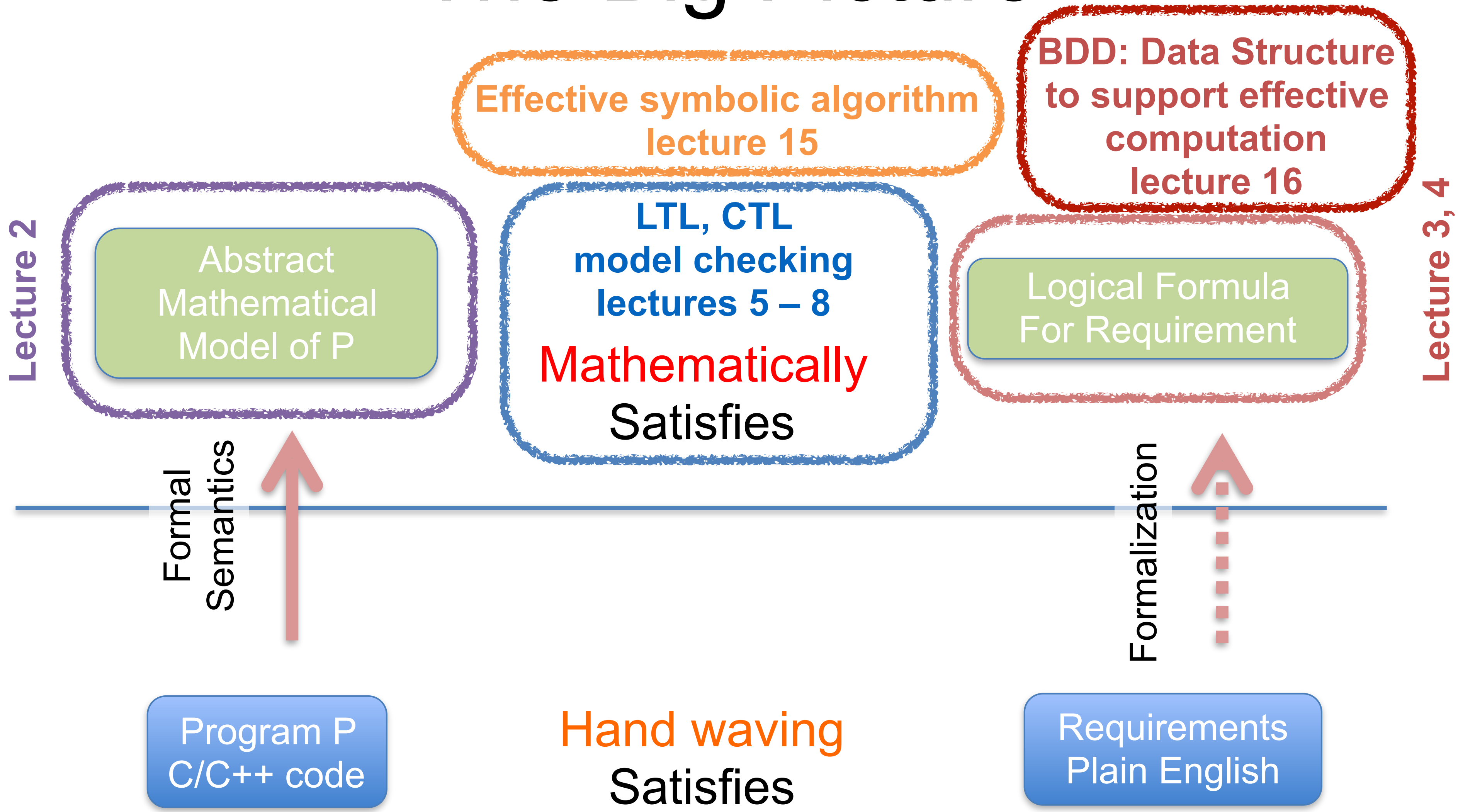
Algorithmic Verification COMP 3153

Lecture 17

Symbolic Model Checking – 2

(last update: April 16, 2018)

The Big Picture



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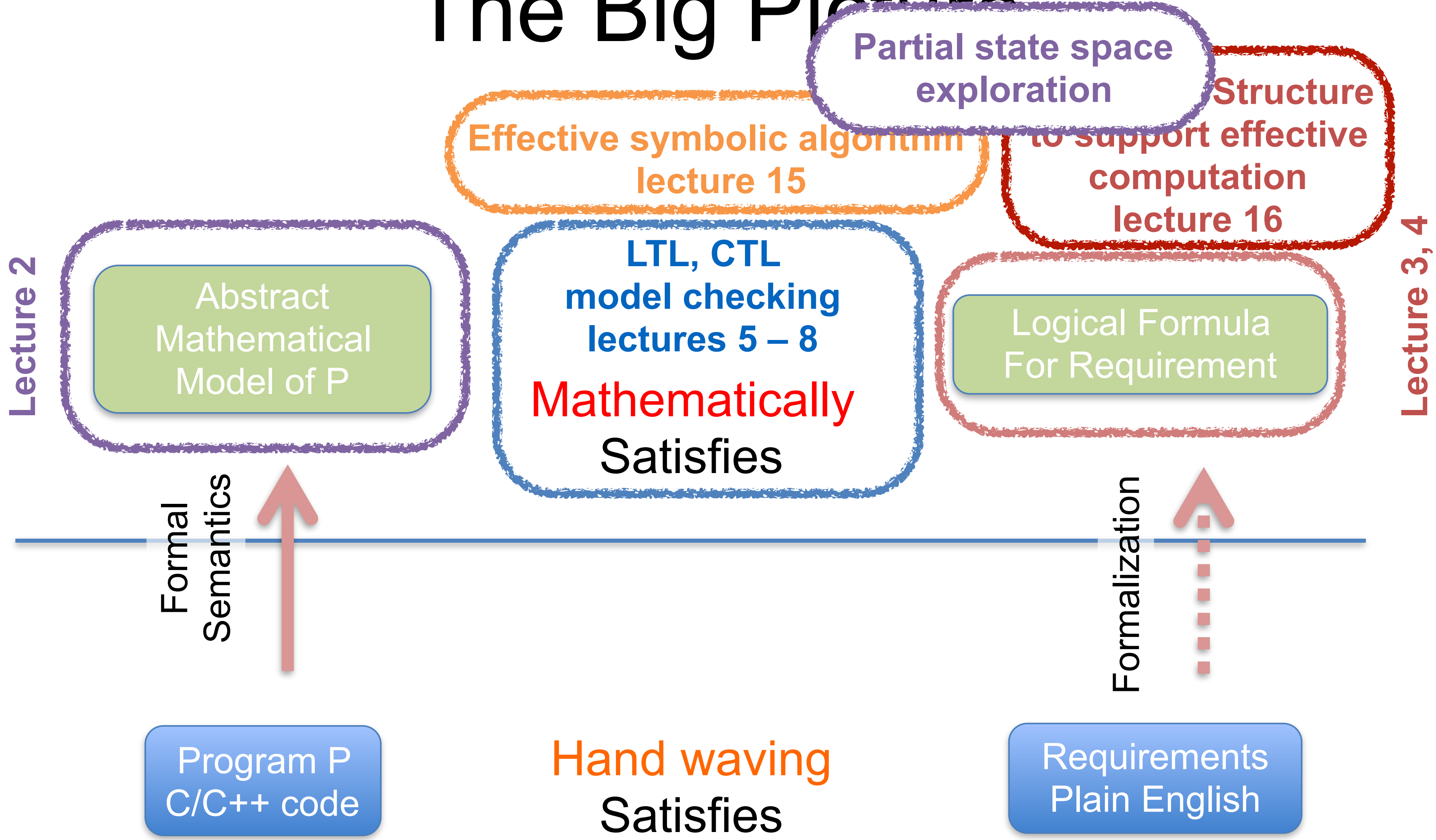


Content for this lecture

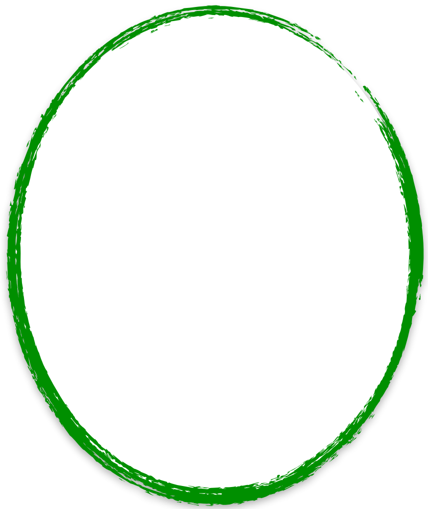
1. SAT problem
2. Model Checking with SAT?
3. Bounded LTL Semantics
4. Bounded Semantics to SAT

SAT Problem

SAT Problem

s vector of ***n*** boolean variables $\boldsymbol{s[1], s[2], \cdots, s[n]}$

Assignment ν : $\{1, 2, \cdots, n\} \rightarrow \{0, 1\}$

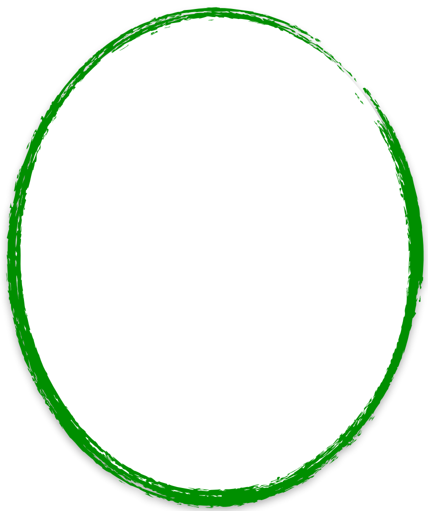


SAT Problem

$\boldsymbol{s}$ vector of $\boldsymbol{n}$ boolean variables $\boldsymbol{s}[1], \boldsymbol{s}[2], \cdots, \boldsymbol{s}[\boldsymbol{n}]$

Assignment $\nu : \{1, 2, \cdots, n\} \to \{0, 1\}$

Set of all assignments: $\mathbb{B}^n$



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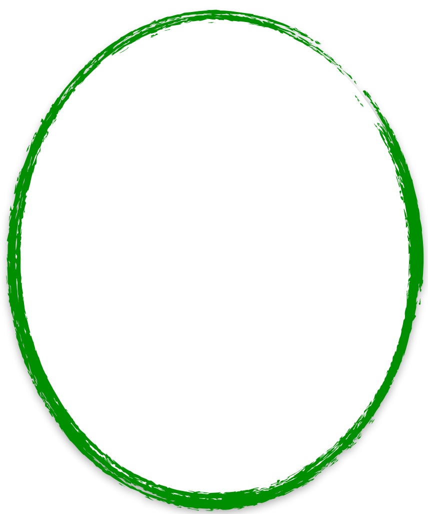

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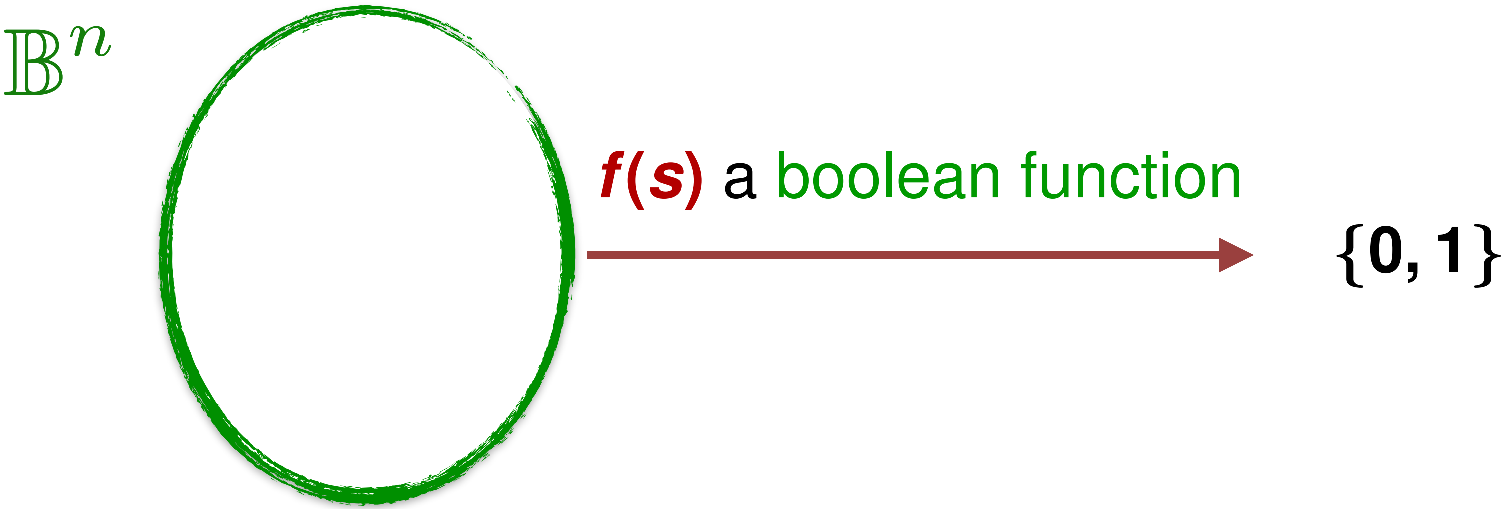

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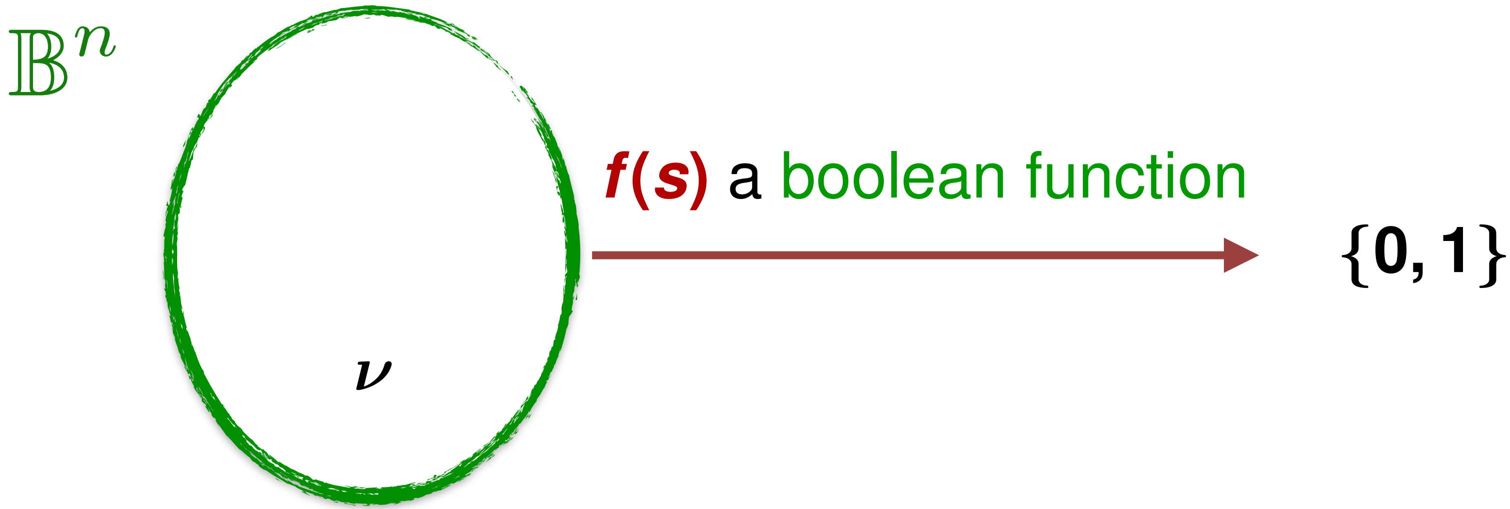

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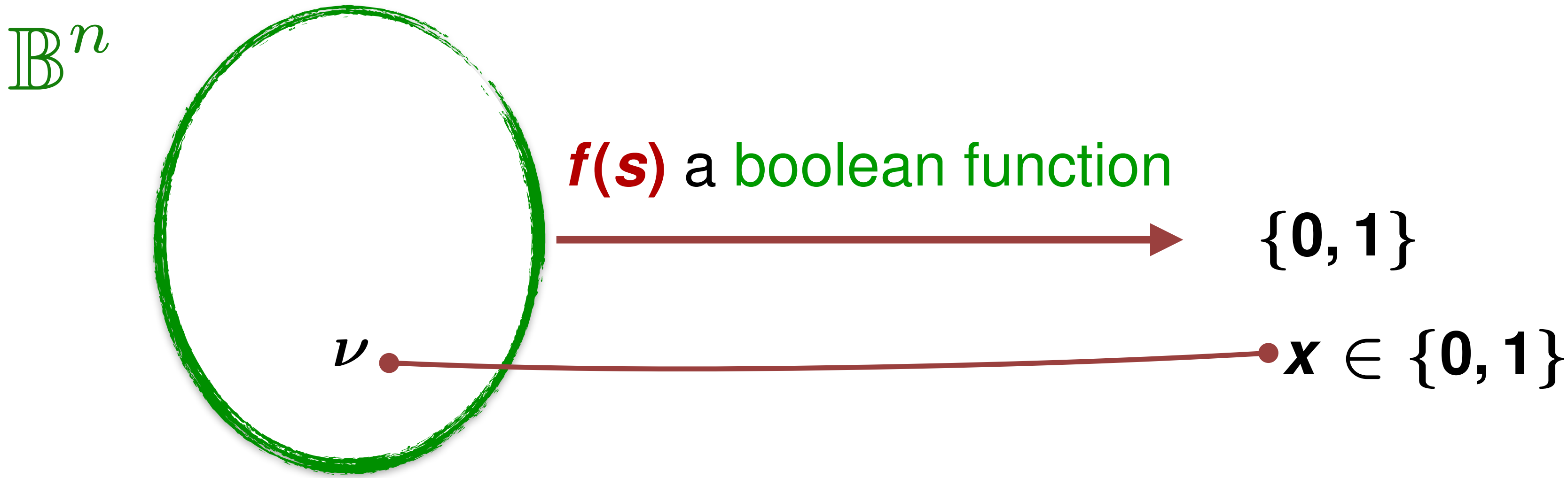

SAT Problem

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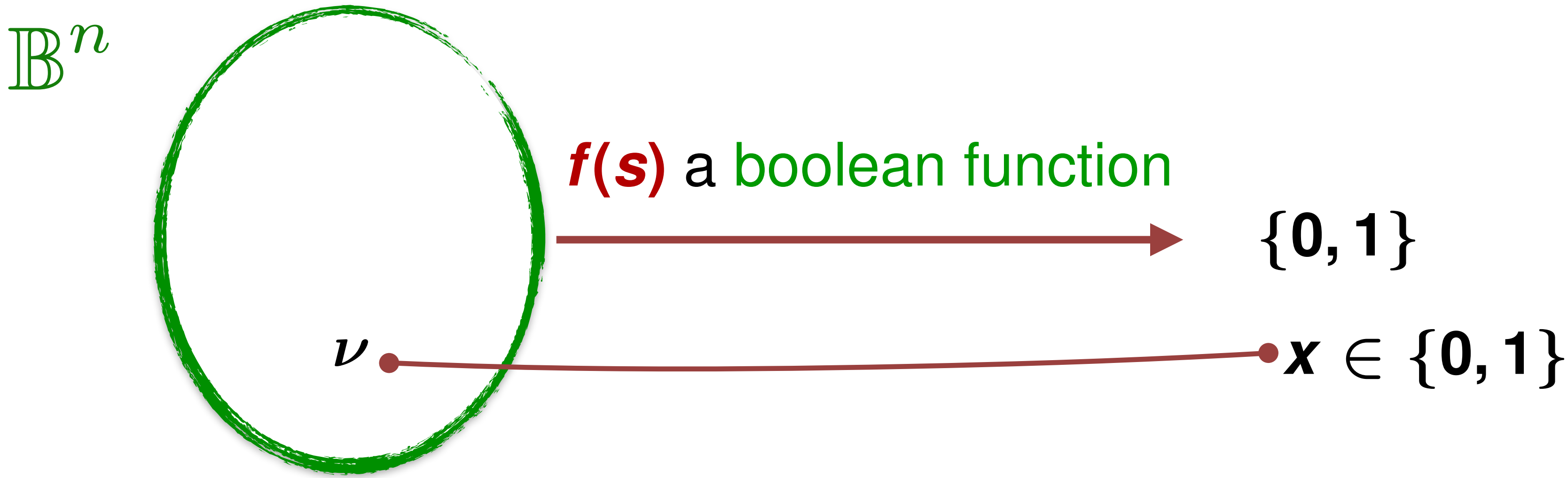

SAT Problem: $\exists \nu \in \mathbb{B}^n$ such that $f(\nu) = 1$?

Complexity of SAT Problem

SAT is NP-complete

Complexity of SAT Problem

SAT is NP-complete

Cook-Levin Theorem, 1971

Complexity of SAT Problem

SAT is NP-complete

Cook-Levin Theorem, 1971

3SAT

- **3** variables
- Formulas in Conjunctive Normal Form

Complexity of SAT Problem

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Cook-Levin Theorem, 1971

3SAT

- **3** variables
- Formulas in Conjunctive Normal Form

3SAT is NP-complete

Karp Theorem, 1971

SAT Solvers in Practise

SAT Solvers in Practise

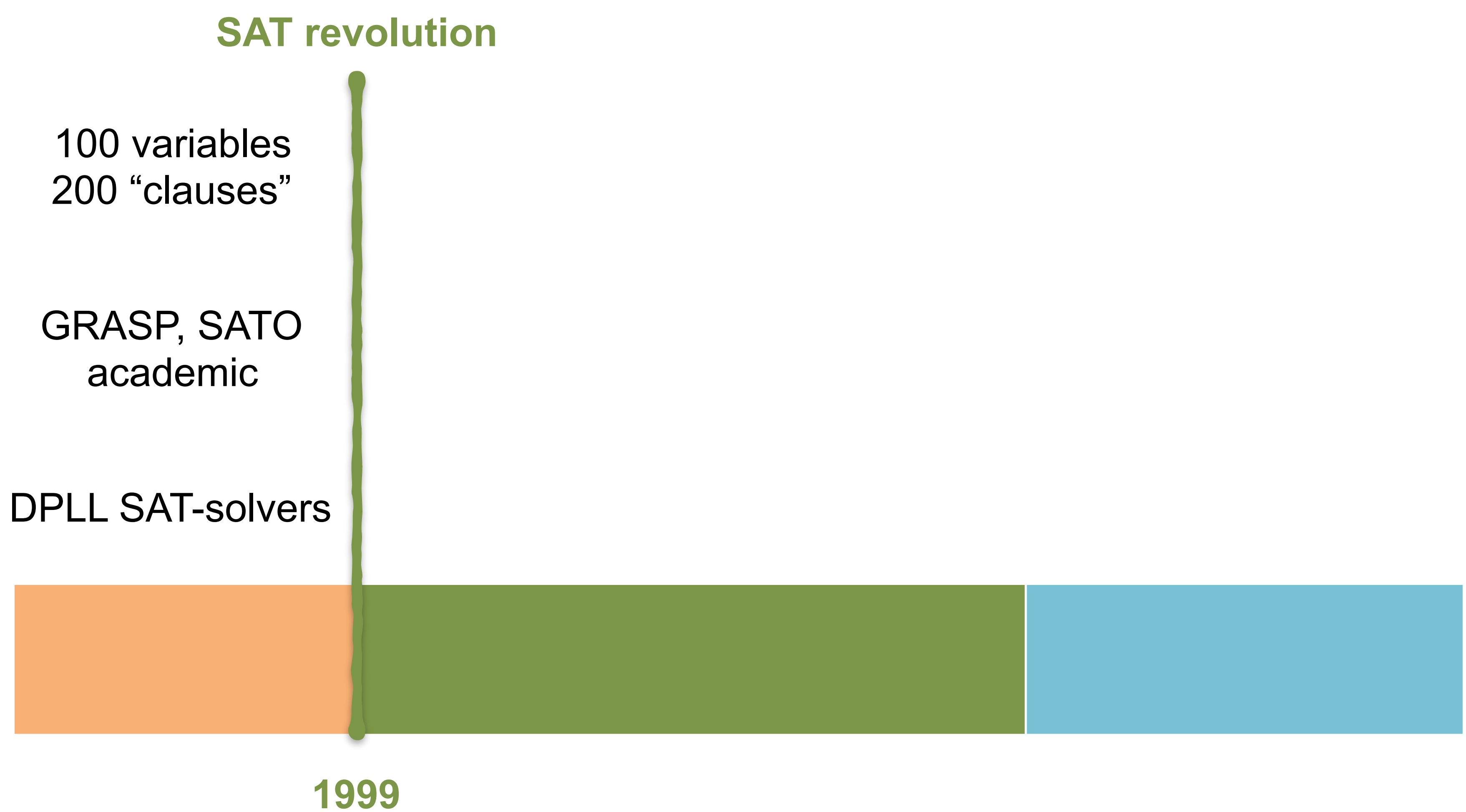
100 variables
200 "clauses"

GRASP, SATO
academic

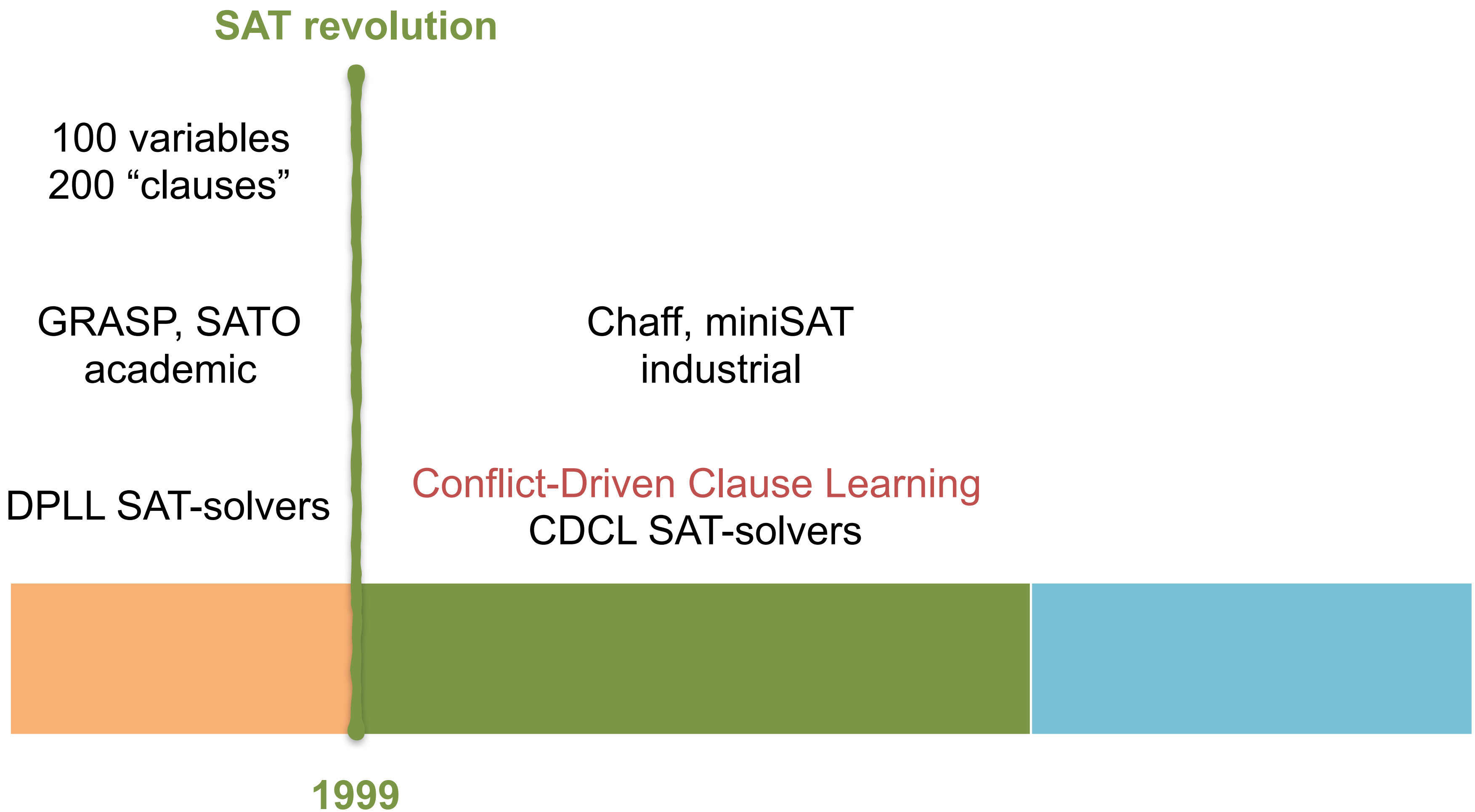
DPLL SAT-solvers

1999

SAT Solvers in Practise



SAT Solvers in Practise



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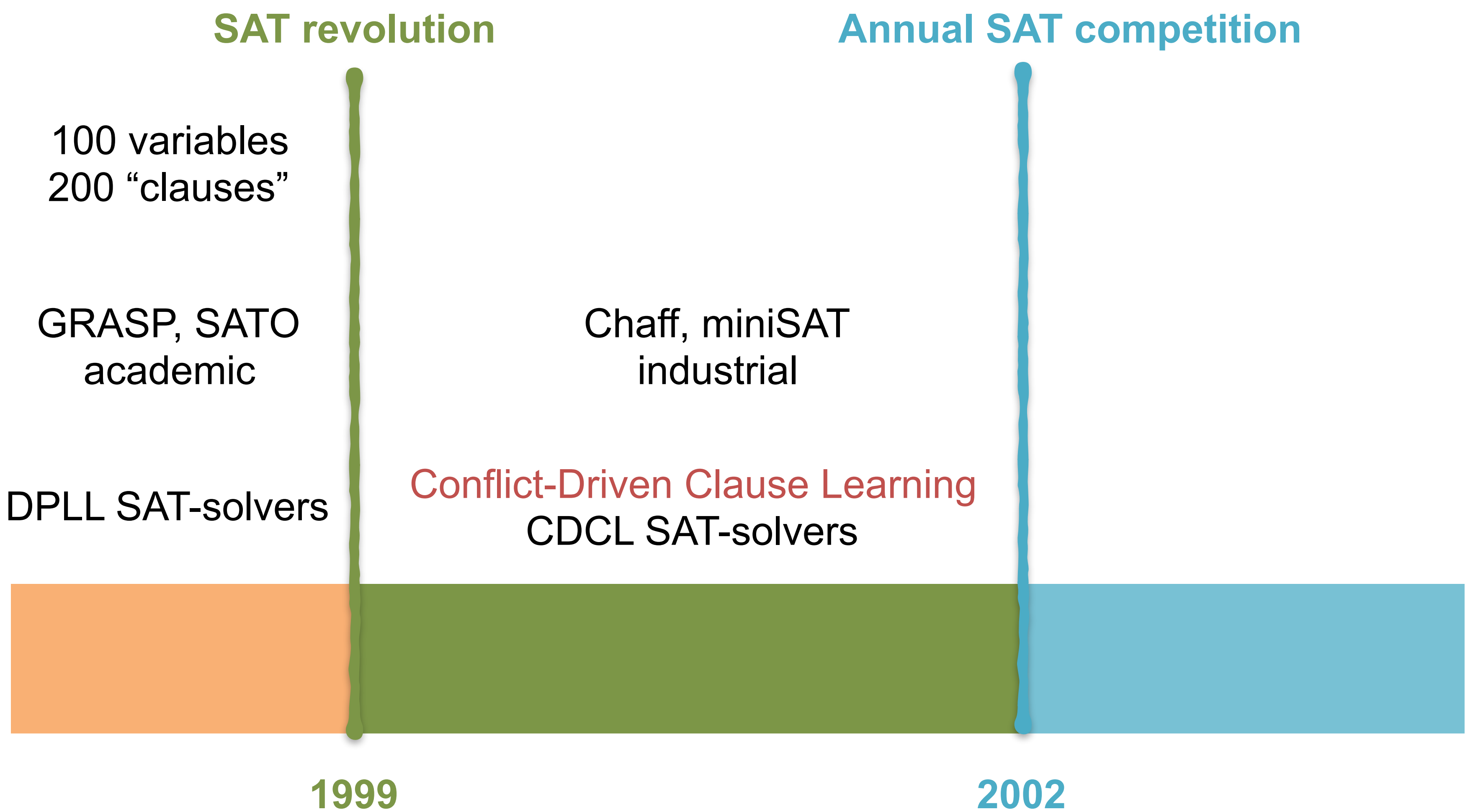

SAT Solvers in Practise

SAT revolution — **1999**

Annual SAT competition — **2002**

100 variables
200 “clauses”

GRASP, SATO
academic

DPLL SAT-solvers

Chaff, miniSAT
industrial

Conflict-Driven Clause Learning
CDCL SAT-solvers

1M variables
5M “clauses”

50+ SAT-solvers
+ SMT-solvers

Model Checking with SAT?

Motivating Example

3-bit Shift Register

Right Shift

1	**1**	**0**

3-bit Shift Register

Right Shift

0	**1**	**1**	**0**

3-bit Shift Register

Right Shift

***x*[2]**	***x*[1]**	***x*[0]**	
0	**1**	**1**	**0**

Store register state in vector ***x***

3-bit Shift Register

Right Shift

***x*[2]**	***x*[1]**	***x*[0]**	
0	**1**	**1**	**0**

Store register state in vector ***x***

Next state ***x′***

3-bit Shift Register

Right Shift

$x[2]$	$x[1]$	$x[0]$	
0	**1**	**1**	**0**

Store register state in vector $\boldsymbol{x}$

Next state $\boldsymbol{x}'$

Transition relation

$$x'[0] = x[1] \land x'[1] = x[2] \land x'[2] = 0$$

3-bit Shift Register

Right Shift

$x[2]$	$x[1]$	$x[0]$	
0	1	1	0

Store register state in vector x

Next state x'

Transition relation

$$x'[0] = x[1] \wedge x'[1] = x[2] \wedge x'[2] = \overset{1}{\cancel{0}}$$

Error in design

3-bit Shift Register

Right Shift

$x[2]$	$x[1]$	$x[0]$	
0	**1**	**1**	**0**

Store register state in vector x

Property: F(x = 0)

Next state x'

Transition relation

$$x'[0] = x[1] \wedge x'[1] = x[2] \wedge x'[2] = \overset{1}{\cancel{0}}$$

Error in design

Strategy

- try to find error/bug
- prove that $\neg\mathrm{F}(\boldsymbol{x} = \mathbf{0})$ holds
  $\mathrm{G}(\boldsymbol{x} \neq \mathbf{0})$ for some path
- look for path with $\leq \boldsymbol{k}$ different states

Paths with less than 3 States

One transition to encode:

$$\boldsymbol{T(x, x')} \stackrel{def}{=} \boldsymbol{x'[0] = x[1] \wedge x'[1] = x[2] \wedge x'[2] = 1}$$

Paths with less than 3 States

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3 vectors: $x_0,\ x_1,\ x_2$

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Paths of length $\leq$ **2**

$$T(x_0, x_1) \wedge T(x_1, x_2)$$

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$x_0[0]$
$x_0[1]$
$x_0[2]$

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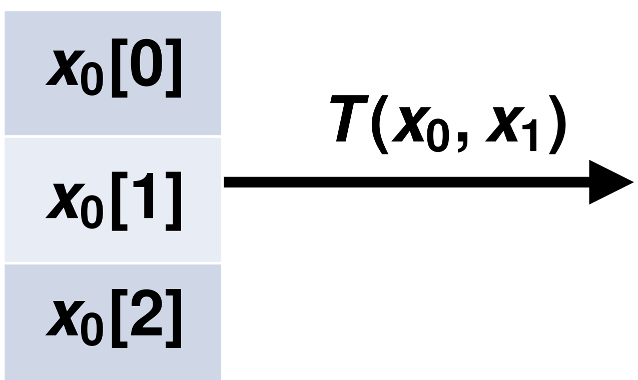

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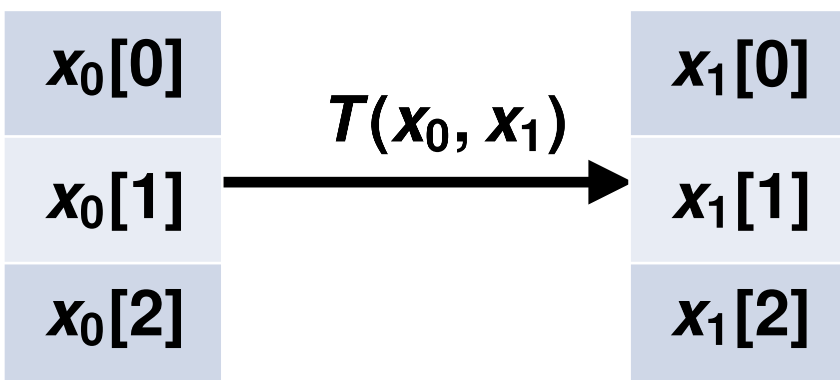
Paths of length $\leq$ **2**

$$T(x_0, x_1) \land T(x_1, x_2)$$



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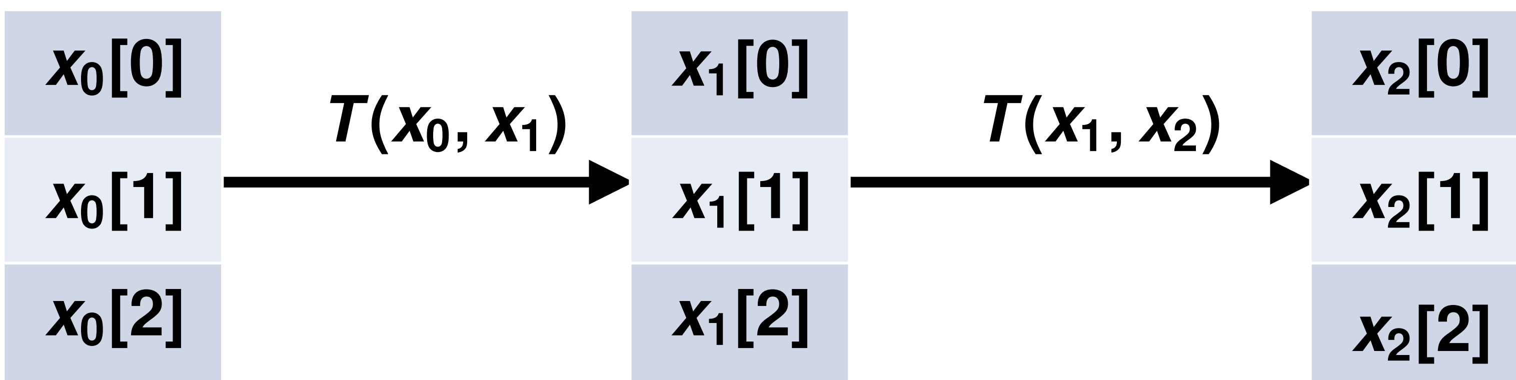
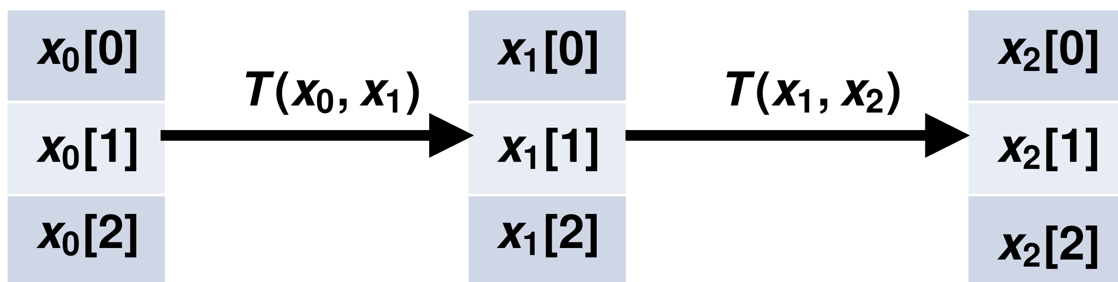


Infinite Paths

$T(x, x') \stackrel{def}{=} x'[0] = x[1] \wedge x'[1] = x[2] \wedge x'[2] = 1$

Goal: find a witness path for $\mathbf{G}(x \neq 0)$



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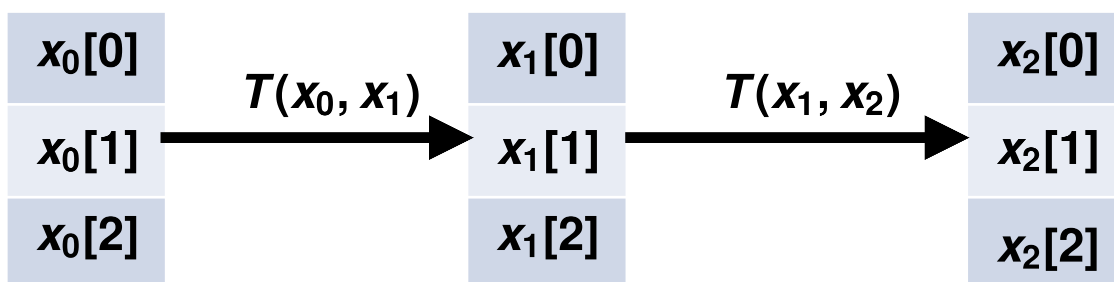
Witness path should be infinite



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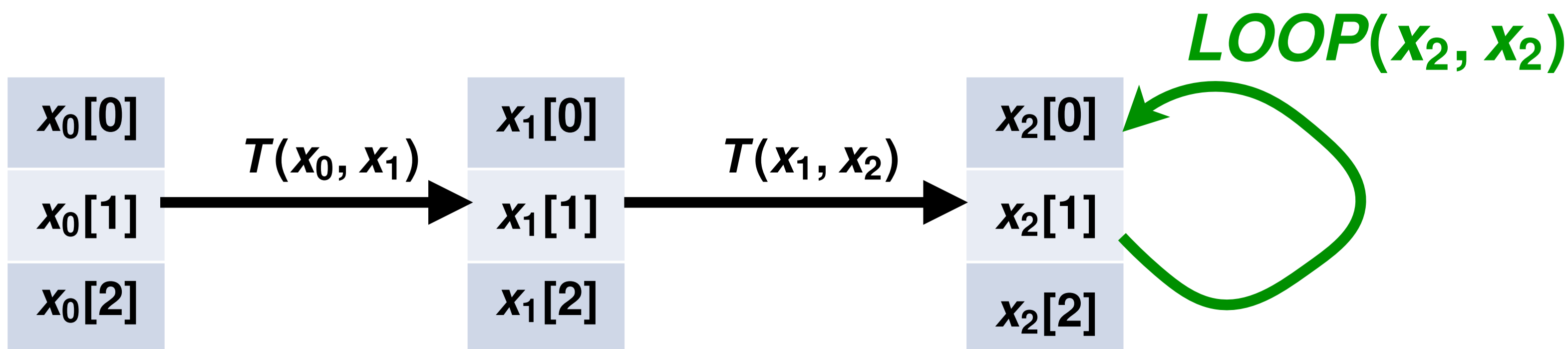

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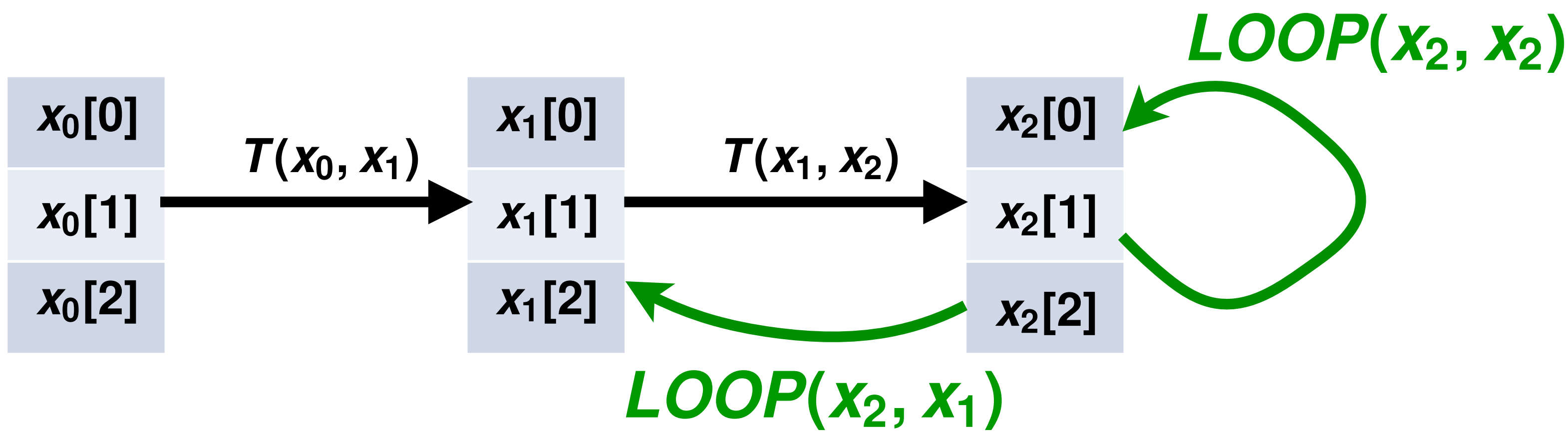

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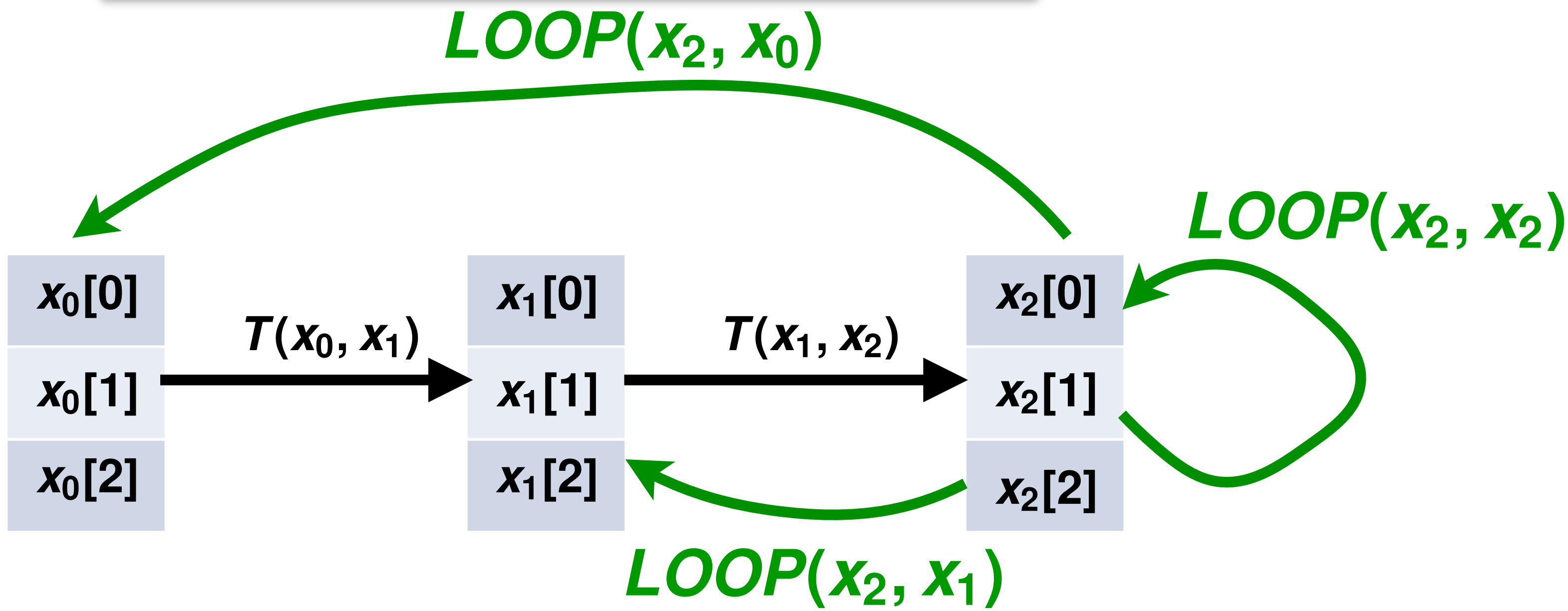

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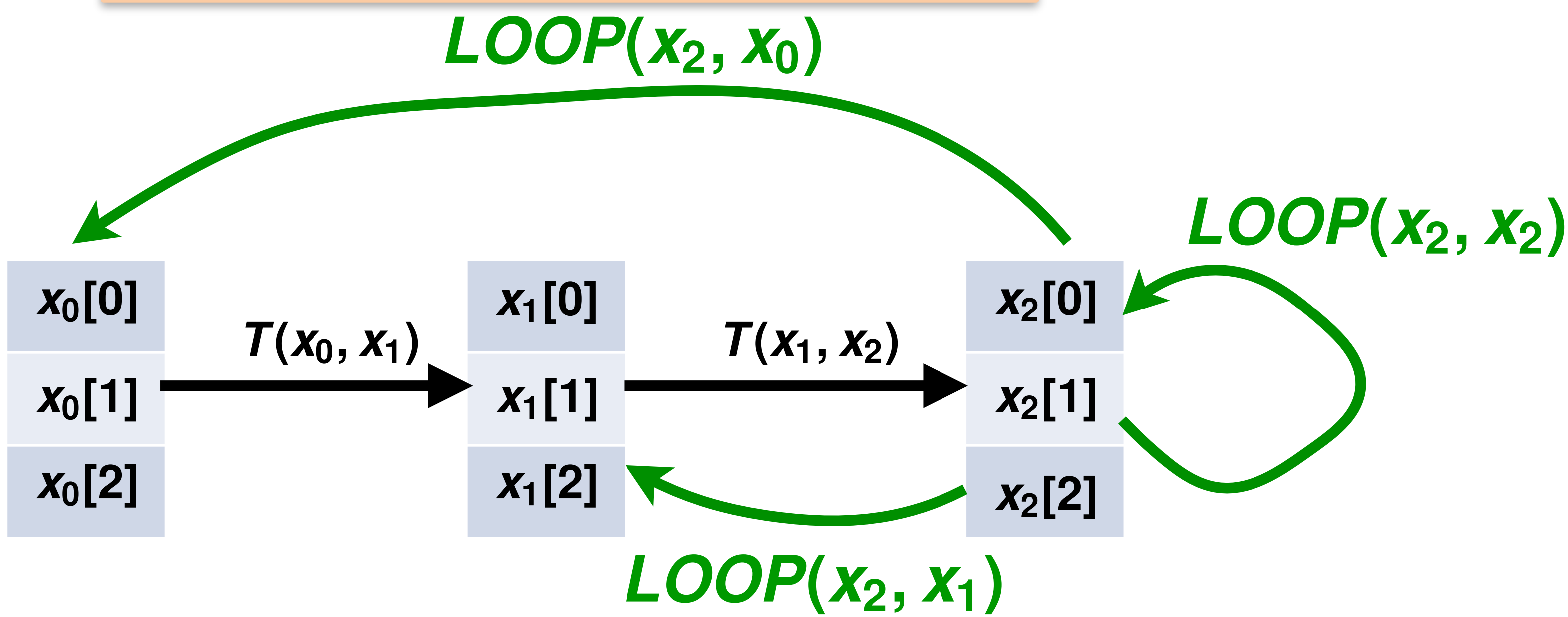

$LOOP(x_2, x_i) = x_i[0] = x_2[1] \wedge x_i[1] = x_2[2] \wedge x_i[2] = 1$

Encoding of 3-State Infinite Paths

$$T(x, x') \stackrel{def}{=} x'[0] = x[1] \wedge x'[1] = x[2] \wedge x'[2] = 1$$

Encoding of 3-State Infinite Paths

$\boldsymbol{T(x, x') \stackrel{def}{=} x'[0] = x[1] \wedge x'[1] = x[2] \wedge x'[2] = 1}$

First two transition: $\boldsymbol{T(x_0, x_1) \wedge T(x_1, x_2)}$

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Loop transitions:

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Paths with less than **3** states:

$$I(x_0) \wedge T(x_0, x_1) \wedge T(x_1, x_2) \wedge \left(\bigvee_{i=0}^{2} LOOP(x_2, x_i)\right)$$

Encoding of 3-State Infinite Paths

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Exercise 1

$$\varphi \stackrel{def}{=} I(x_0) \wedge T(x_0, x_1) \wedge T(x_1, x_2) \wedge \left(\bigvee_{i=0}^{2} LOOP(x_2, x_i)\right)$$

- Is the following path ϱ allowed by φ?

  $\varrho = x_0\ x_1\ x_2\ x_1\ x_2\ x_0\ x_1\ x_2 \cdots$

- What are the infinite paths specified by φ?

Encoding of G($\boldsymbol{x} \neq \mathbf{0}$)

$$x \neq 0 \iff \boxed{x[0] = 1 \lor x[1] = 1 \lor x[2] = 1}$$

Encoding of G($\boldsymbol{x \neq 0}$)

$$x \neq 0 \iff \underbrace{\boxed{x[0] = 1 \lor x[1] = 1 \lor x[2] = 1}}_{\psi(x)}$$

Encoding of G($\boldsymbol{x} \neq \mathbf{0}$)

$$\boldsymbol{x} \neq \mathbf{0} \iff \underbrace{\boxed{\boldsymbol{x}[0] = 1 \vee \boldsymbol{x}[1] = 1 \vee \boldsymbol{x}[2] = 1}}_{\psi(\boldsymbol{x})}$$

$$\boxed{\mathrm{G}(\boldsymbol{x} \neq \mathbf{0}) \stackrel{def}{=} \bigwedge_{i=0}^{2} \psi(\boldsymbol{x}_i)}$$

Encoding of $G(x \neq 0)$

$$x \neq 0 \iff \underbrace{\boxed{x[0] = 1 \lor x[1] = 1 \lor x[2] = 1}}_{\psi(x)}$$

$$\boxed{G(x \neq 0) \stackrel{def}{=} \bigwedge_{i=0}^{2} \psi(x_i)}$$

Paths satisfying $G(x \neq 0)$

$$\boxed{I(x_0) \land T(x_0, x_1) \land T(x_1, x_2) \land \left(\bigvee_{i=0}^{2} LOOP(x_2, x_i)\right) \land \bigwedge_{i=0}^{2} \psi(x_i)}$$

Reduction to SAT

Reduction to SAT

There exists an infinite path with less than **3** states that satisfy G$(\boldsymbol{x} \neq \mathbf{0})$

Reduction to SAT

There **exists** an **infinite path** with less than **3** states that satisfy $G(\boldsymbol{x} \neq \mathbf{0})$

$$\Longleftrightarrow$$

Reduction to SAT

There exists an infinite path with less than **3** states that satisfy G($\boldsymbol{x} \neq \mathbf{0}$)

$$\Longleftrightarrow$$

$\exists \boldsymbol{x}_0, \boldsymbol{x}_1, \boldsymbol{x}_2$ such that

$$\boldsymbol{I}(\boldsymbol{x}_0) \wedge \boldsymbol{T}(\boldsymbol{x}_0, \boldsymbol{x}_1) \wedge \boldsymbol{T}(\boldsymbol{x}_1, \boldsymbol{x}_2) \wedge \left(\bigvee_{i=0}^{2} \boldsymbol{LOOP}(\boldsymbol{x}_2, \boldsymbol{x}_i)\right) \wedge \bigwedge_{i=0}^{2} \psi(\boldsymbol{x}_i)$$

is SATisfiable

Reduction to SAT

There **exists** an **infinite path** with less than **3** states that satisfy $G(x \neq 0)$

$$\Longleftrightarrow$$

$\exists x_0, x_1, x_2$ such that

$$\varphi \quad I(x_0) \wedge T(x_0, x_1) \wedge T(x_1, x_2) \wedge \left(\bigvee_{i=0}^{2} LOOP(x_2, x_i)\right) \wedge \bigwedge_{i=0}^{2} \psi(x_i)$$

is SATisfiable

Reduction to SAT

There exists an infinite path with less than **3** states that satisfy $\mathrm{G}(\boldsymbol{x} \neq \mathbf{0})$

$\Longleftrightarrow$

Run SAT-solver on φ: SAT!

$\exists \boldsymbol{x}_0, \boldsymbol{x}_1, \boldsymbol{x}_2$ such that

$$\varphi \quad \boldsymbol{I}(\boldsymbol{x}_0) \wedge \boldsymbol{T}(\boldsymbol{x}_0, \boldsymbol{x}_1) \wedge \boldsymbol{T}(\boldsymbol{x}_1, \boldsymbol{x}_2) \wedge \left(\bigvee_{i=0}^{2} \boldsymbol{LOOP}(\boldsymbol{x}_2, \boldsymbol{x}_i)\right) \wedge \bigwedge_{i=0}^{2} \psi(\boldsymbol{x}_i)$$

is SATisfiable

Bounded LTL Semantics

Existential Model Checking Problem

LTL model checking problem

Given $\boldsymbol{A}$, labeled automaton, $\boldsymbol{\Phi}$ LTL formula

$$\boldsymbol{A} \models \boldsymbol{\Phi} \iff \forall \rho \in \boldsymbol{Runs(A)},\ \rho \models \boldsymbol{\Phi}$$

Existential Model Checking Problem

LTL model checking problem

Given $\boldsymbol{A}$, labeled automaton, $\boldsymbol{\Phi}$ LTL formula

$$\boldsymbol{A} \models \boldsymbol{\Phi} \iff \forall \rho \in \boldsymbol{Runs(A)}, \rho \models \boldsymbol{\Phi}$$

LTL Universal Validity

$$\boldsymbol{A} \models \mathrm{A}\boldsymbol{\Phi}$$

$$\iff$$

$$\forall \rho \in \boldsymbol{Runs(A)}, \rho \models \boldsymbol{\Phi}$$

Existential Model Checking Problem

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LTL Existential Validity

$$\boldsymbol{A} \models \mathrm{E}\boldsymbol{\Phi}$$
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Existential Model Checking Problem

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LTL Existential Validity

$$\boldsymbol{A} \models \mathrm{E}\boldsymbol{\Phi}$$
$$\iff$$
$$\exists \rho \in \boldsymbol{Runs}(\boldsymbol{A}),\ \rho \models \boldsymbol{\Phi}$$

$$\boldsymbol{A} \models \mathrm{A}\boldsymbol{\Phi} \iff \boldsymbol{A} \not\models \mathrm{E}\neg\boldsymbol{\Phi}$$

LTL Formulas – Semantics

Given a run $\rho = s_0 \cdots s_n \cdots$

Notation: $\rho[i] = s_i \cdots s_n \cdots, \; i \geq 0$

P propositional formula, φ_1, φ_2 LTL formulas

- $\rho \models P \iff s_0 \models P$
- Next: $\rho \models X\varphi_1 \iff \rho[1] \models \varphi_1$
- Until: $\rho \models \varphi_1 \text{ UNTIL } \varphi_2 \iff \begin{cases} \exists k \geq 0, \rho[k] \models \varphi_2 \\ \forall 0 \leq j < k, \rho[j] \models \varphi_1 \end{cases}$
- Eventually: $\rho \models \mathbf{F}\,\varphi_1 \iff \exists k \geq 0, \rho[k] \models \varphi_1$
- Globally: $\rho \models \mathbf{G}\,\varphi_1 \iff \forall k \geq 0, \rho[k] \models \varphi_1$

Negation Normal Form

Negation Normal Form = $\neg$ only allowed on atomic propositions

Negation Normal Form

Negation Normal Form = $\neg$ only allowed on atomic propositions

Rewrite rules

$$\neg(\varphi_1 \wedge \varphi_2) \equiv (\neg\varphi_1) \vee (\neg\varphi_2)$$

$$\neg \mathrm{F}\varphi \equiv \mathrm{G}\neg\varphi$$

$$\neg \mathrm{X}\varphi \equiv \mathrm{X}\neg\varphi$$

$$\neg \mathrm{G}\varphi \equiv \mathrm{F}\neg\varphi$$

Negation Normal Form

Negation Normal Form = $\neg$ only allowed on atomic propositions

Rewrite rules

$$\neg(\varphi_1 \wedge \varphi_2) \equiv (\neg\varphi_1) \vee (\neg\varphi_2) \qquad \neg F\varphi \equiv G\neg\varphi$$

$$\neg X\varphi \equiv X\neg\varphi \qquad \neg G\varphi \equiv F\neg\varphi$$

From now on: Every LTL formula in NNF

- $\neg$ only atomic propositions
- $\vee$ allowed (semantics easy to write)

Bounded Semantics

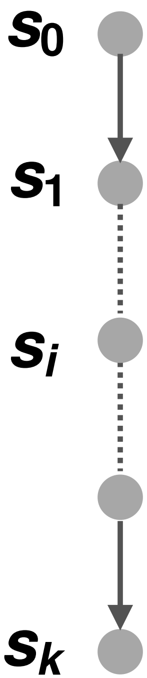
Subset of all paths in $\boldsymbol{A}$

$\leq \boldsymbol{k+1}$ states

Bounded Semantics

Subset of all paths in $\boldsymbol{A}$

$\leq \boldsymbol{k+1}$ states

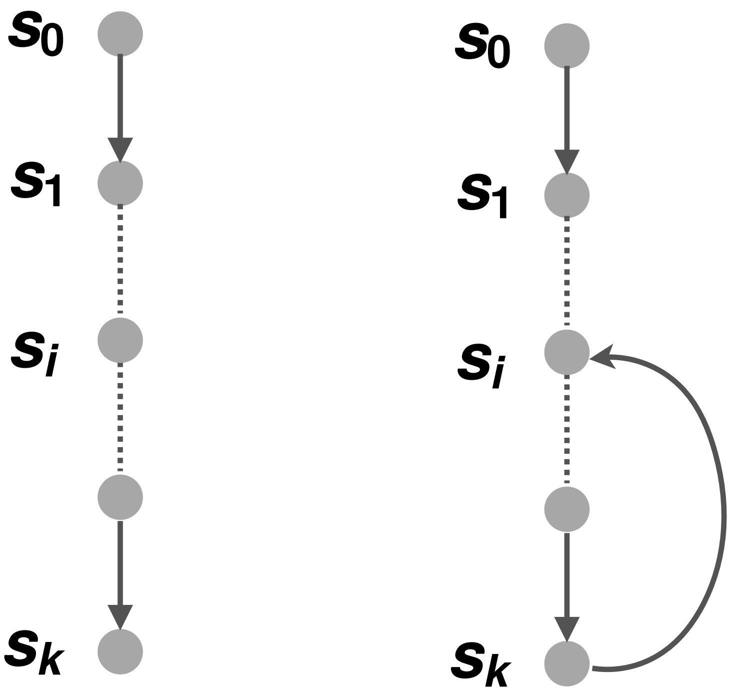


Finite paths

Bounded Semantics

Subset of all paths in $\boldsymbol{A}$

$\leq$ $\boldsymbol{k}$ **+ 1** states



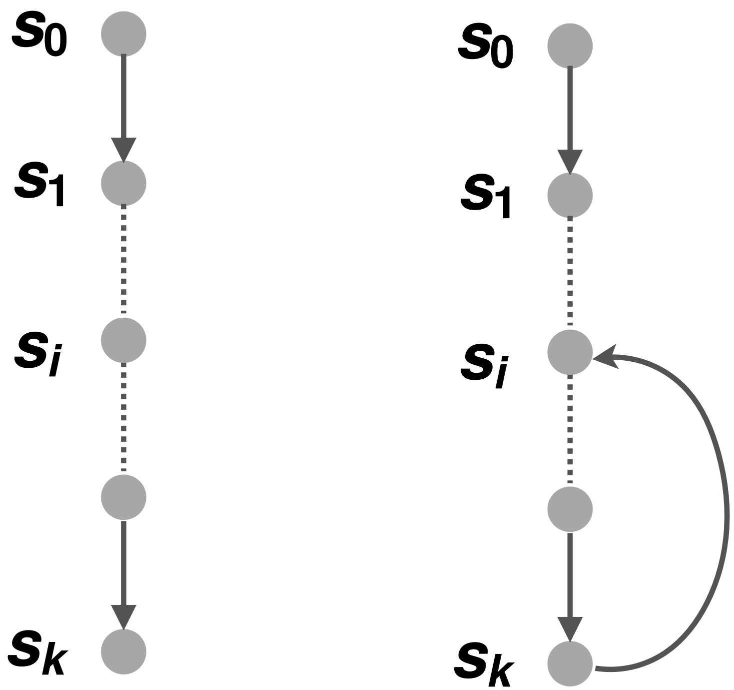
(k, i)-loop

Finite paths Lasso infinite paths

Bounded Semantics

Subset of all paths in $\boldsymbol{A}$

$\leq \boldsymbol{k+1}$ states



(*k*, *i*)-loop

Finite paths   Lasso infinite paths

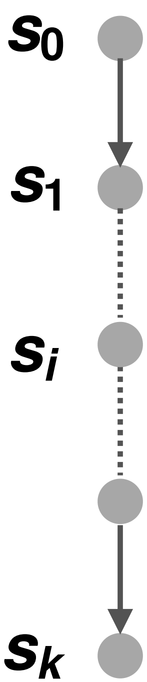
Bounded Semantics of LTL

$$\rho \models_k \varphi \implies \rho \models \varphi$$

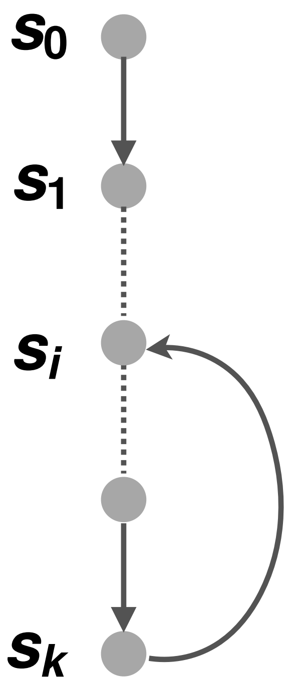
Bounded Semantics

Subset of all paths in $\boldsymbol{A}$

$\leq k + 1$ states



Finite paths



(k, i)-loop

Lasso infinite paths

Bounded Semantics of LTL

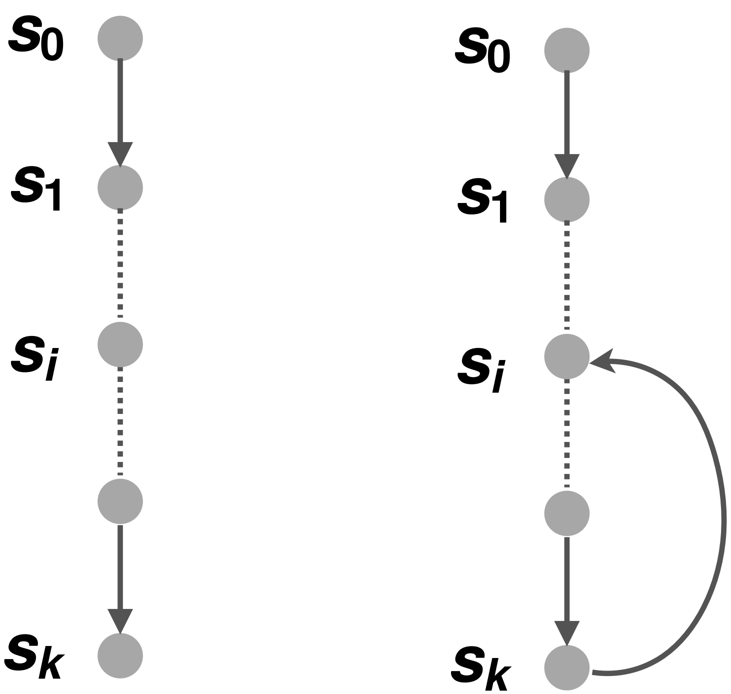
$$\rho \models_k \varphi \implies \rho \models \varphi$$

Soundness

Bounded Semantics

Subset of all paths in A

$\leq k + 1$ states



(k, i)-loop

Finite paths Lasso infinite paths

Bounded Semantics of LTL

$$\rho \models_k \varphi \implies \rho \models \varphi$$

Soundness

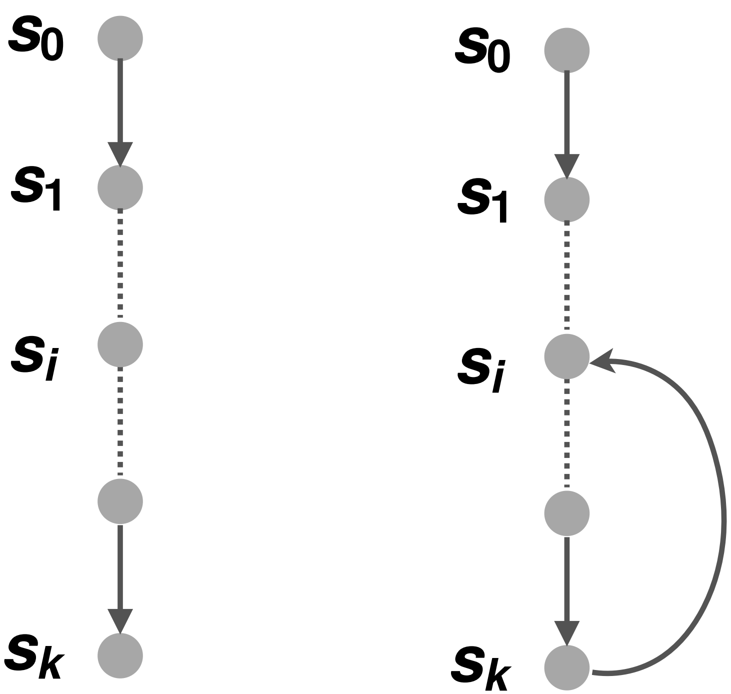
k-loop path ρ

$\exists l$ ρ is a (k, l)-loop

Bounded Semantics

Subset of all paths in ***A***

$\leq$ ***k* + 1** states



(***k*, *i***)-loop

Finite paths Lasso infinite paths

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$$\rho \models_k \varphi \implies \rho \models \varphi$$

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$\exists$***l*** ρ is a (***k*, *l***)-loop

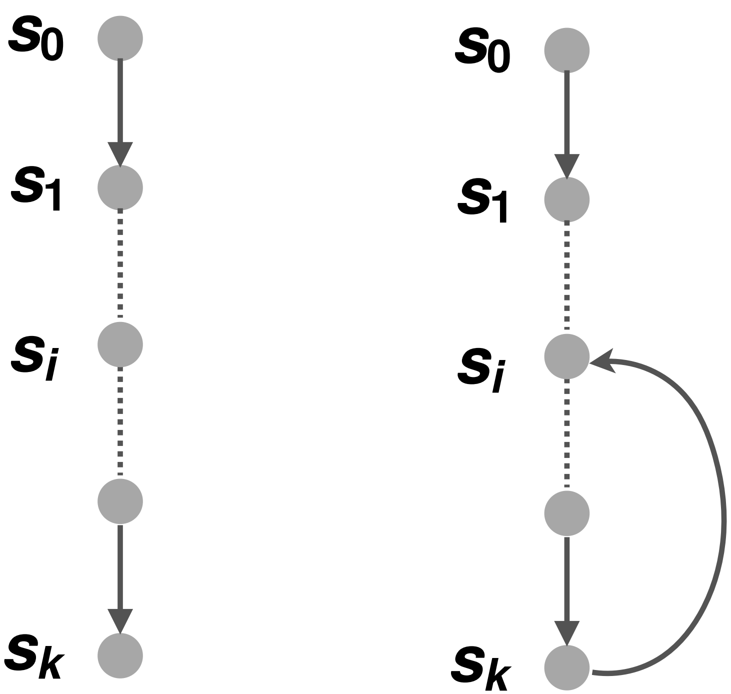
ρ is a ***k***-loop

$$\rho \models_k \varphi \iff \rho \models \varphi$$

Bounded Semantics

Subset of all paths in A

$\leq k + 1$ states



(k, i)-loop

Finite paths   Lasso infinite paths

Bounded Semantics of LTL

$$\rho \models_k \varphi \implies \rho \models \varphi$$

Soundness

k-loop path ρ

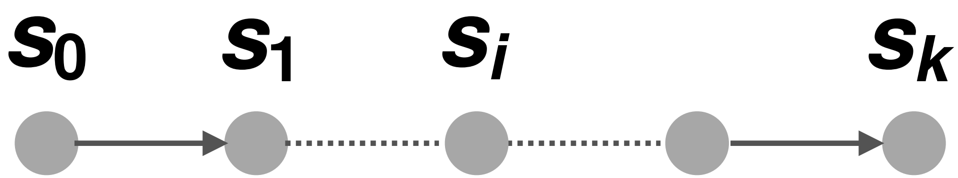
$\exists l\ \rho$ is a (k, l)-loop

ρ is a k-loop

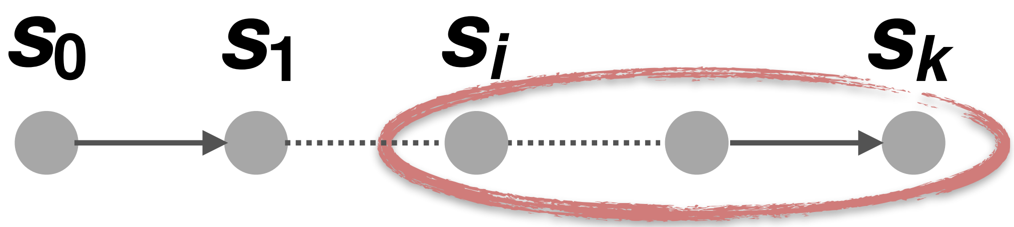
$$\rho \models_k \varphi \iff \rho \models \varphi$$

What about finite paths?

Bounded Length Paths – 1

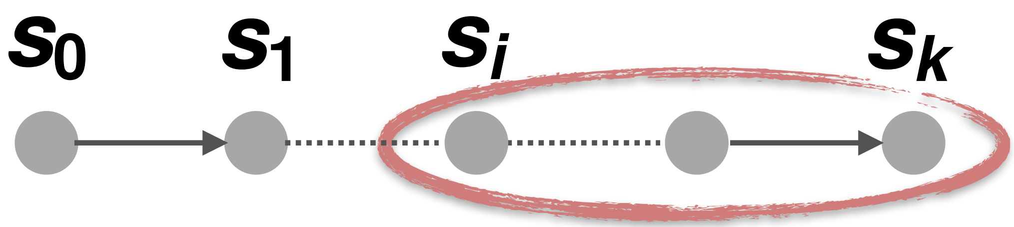


Bounded Length Paths – 1



$0 \leq i \leq k, \models^i_k$ satisfaction relation on subpath $s_i \cdots s_k$

Bounded Length Paths – 1



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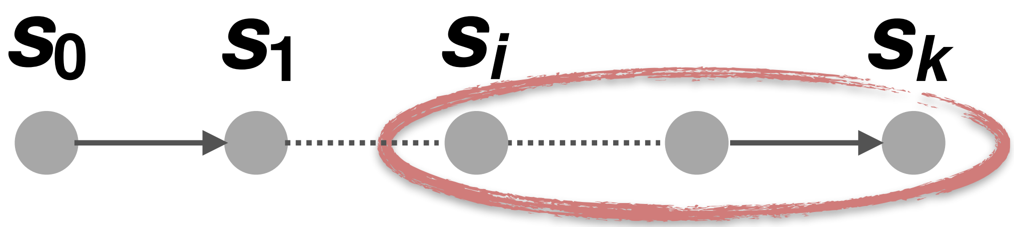
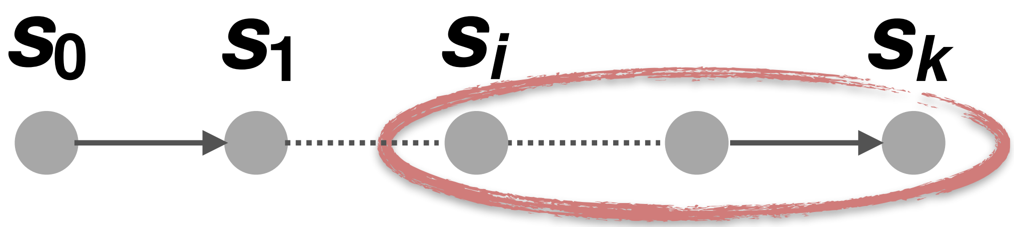


$0 \leq i \leq k$, $\models^i_k$ satisfaction relation on subpath $s_i \cdots s_k$

- $P(s)$ propositional formula: $\begin{cases} \rho \models^i_k P(s) \iff P(s_i) \\ \rho \models^i_k \neg P(s) \iff \neg P(s_i) \end{cases}$
- $\rho \models^i_k \varphi_1 \wedge \varphi_2 \iff \rho \models^i_k \varphi_1$ and $\rho \models^i_k \varphi_2$
- $\rho \models^i_k \mathrm{X}\varphi \iff 0 \leq i < k$ and $\rho \models^{i+1}_k \varphi_1$

Bounded Length Paths – 2



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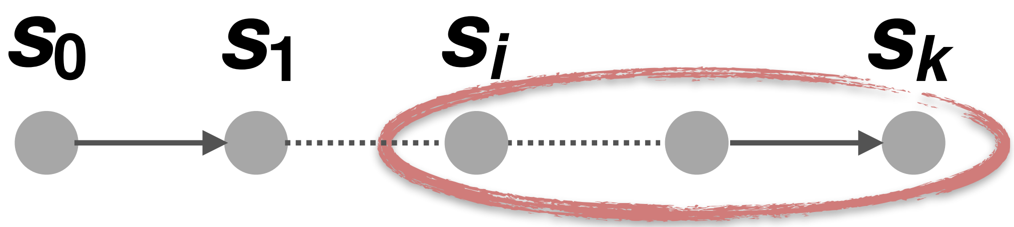
Bounded Length Paths – 2



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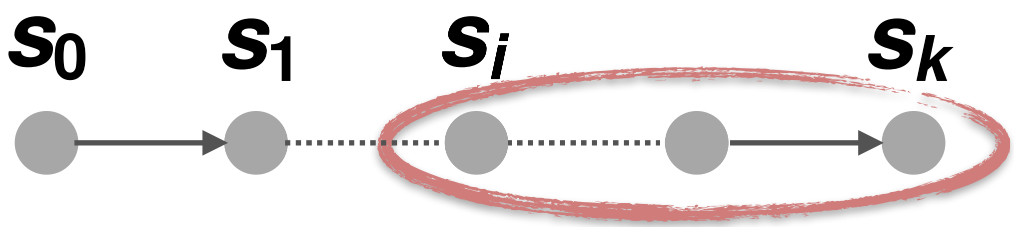
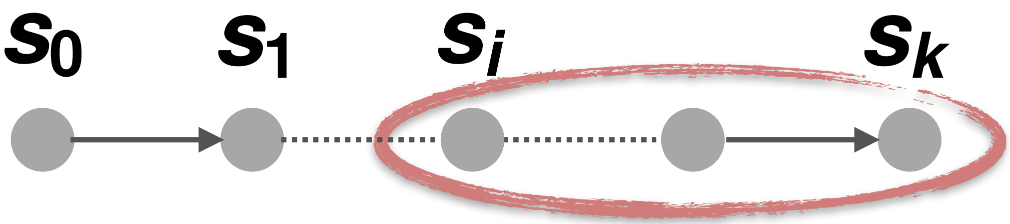
Bounded Length Paths – 2



$0 \leq i \leq k, \models^i_k$ satisfaction relation on subpath $s_i \cdots s_k$

- $\rho \models^i_k \varphi_1$ UNTIL $\varphi_2 \iff \begin{cases} \rho \models^i_k \exists i \leq j \leq k, \rho \models^j_k \varphi_2 \\ \forall i \leq n < j, \rho \models^n_k \varphi_1 \end{cases}$
- $\rho \models^i_k \mathrm{F}\,\varphi \iff \exists i \leq j \leq k, \rho \models^j_k \varphi$
- $\rho \models^i_k \mathrm{G}\,\varphi \iff \textit{False}$

Bounded Length Paths – 2



$\rho \models_k \varphi \iff \rho \models_k^0 \varphi$

$0 \leq i \leq k, \models_k^i$ satisfaction relation on subpath $s_i \cdots s_k$

- $\rho \models_k^i \varphi_1 \text{ UNTIL } \varphi_2 \iff \begin{cases} \rho \models_k^i \exists i \leq j \leq k, \rho \models_k^j \varphi_2 \\ \forall i \leq n < j, \rho \models_k^n \varphi_1 \end{cases}$
- $\rho \models_k^i \mathrm{F}\, \varphi \iff \exists i \leq j \leq k, \rho \models_k^j \varphi$
- $\rho \models_k^i \mathrm{G}\, \varphi \iff \textit{False}$

Exercise 2

LTL semantics

$$\rho \models \neg \mathrm{F} \neg \varphi \iff \rho \models \mathrm{G} \varphi$$

LTL bounded semantics

Is it true that $\rho \models_k \neg \mathrm{F} \neg \varphi \iff \rho \models_k \mathrm{G} \varphi$?

Bounded Semantics to SAT

Objective

$$\boldsymbol{A} \models_{\boldsymbol{k}} \mathrm{E}\varphi \iff \boldsymbol{For}(\boldsymbol{A}) \wedge \boldsymbol{For}(\varphi)_{\boldsymbol{k}} \text{ is SAT}$$

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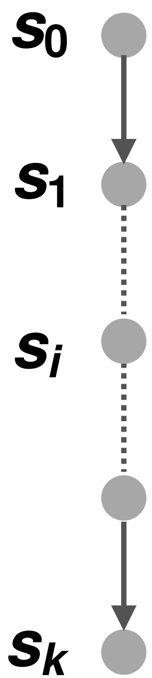


Finite paths

Objective

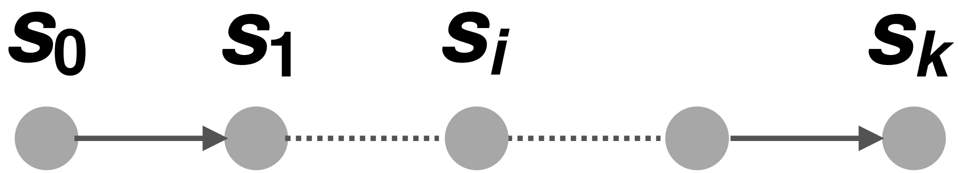
$$\boldsymbol{A} \models_k \mathrm{E}\varphi \iff \boldsymbol{For}(\boldsymbol{A}) \wedge \boldsymbol{For}(\varphi)_k \text{ is SAT}$$

$(\boldsymbol{k}, \boldsymbol{i})$-loop

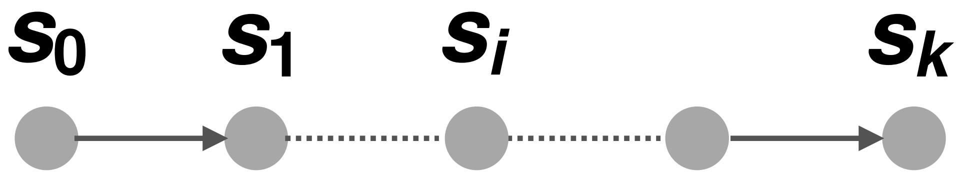
Finite paths

Lasso infinite paths

Finite Paths – 1

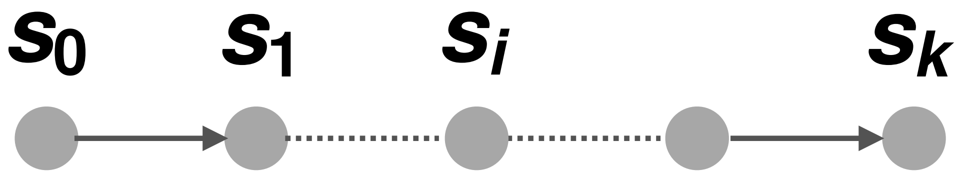


Finite Paths – 1



k fixed $\Longrightarrow$ ***For***(φ)

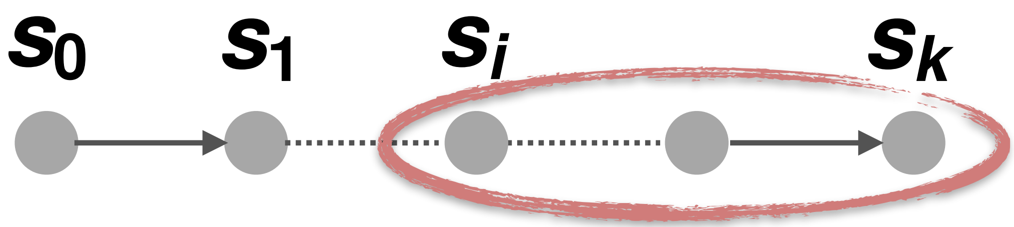
Finite Paths – 1



k fixed $\Longrightarrow$ $\boldsymbol{For}(\varphi)$

$\rho \models_k \varphi \iff \boldsymbol{For}(\varphi)$ is SAT

Finite Paths – 1

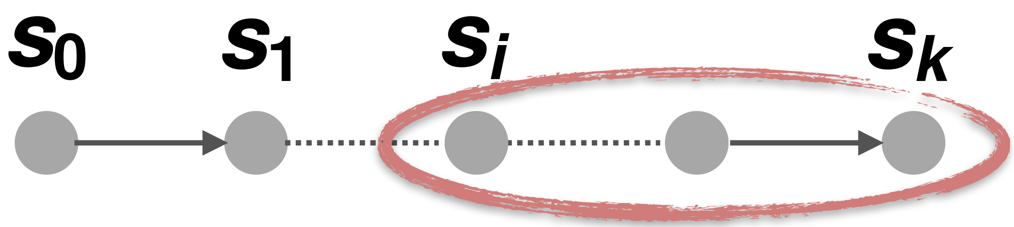


$\boldsymbol{k}$ fixed $\Longrightarrow$ $\boldsymbol{For}(\varphi)$

$\rho \models_k \varphi \iff \boldsymbol{For}(\varphi)$ is SAT

$\rho[i] \models_k \varphi \iff \boldsymbol{For}(\varphi)_i$ is SAT

Finite Paths – 1

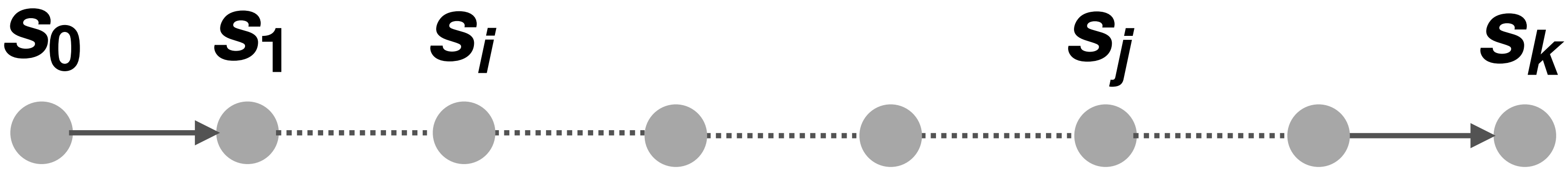


k fixed $\Longrightarrow$ $\mathit{For}(\varphi)$

$\rho \models_k \varphi \iff \mathit{For}(\varphi)$ is SAT $\qquad \rho[i] \models_k \varphi \iff \mathit{For}(\varphi)_i$ is SAT

- $P(s)$ propositional formula: $\mathit{For}(P(s))_i = P(s_i)$
- $\mathit{For}(\varphi_1 \wedge \varphi_2)_i = \mathit{For}(\varphi_1)_i \wedge \mathit{For}(\varphi_2)_i$
- $\mathit{For}(\mathrm{X}\varphi)_i =$ If $0 \leq i < k$ then $\mathit{For}(\varphi)_{i+1}$ else False
- $\mathit{For}(\mathrm{G}\varphi)_i = \mathit{False}$

Finite Paths – 2

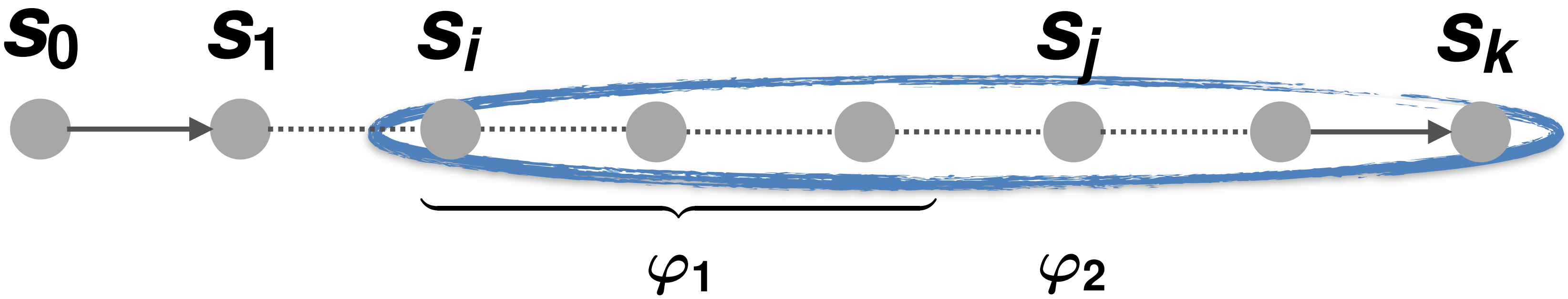


$\boldsymbol{k}$ fixed $\Longrightarrow$ $\boldsymbol{For}(\varphi)$

$\rho \models_k \varphi \iff \boldsymbol{For}(\varphi)$ is SAT $\qquad$ $\rho[i] \models_k \varphi \iff \boldsymbol{For}(\varphi)_i$ is SAT

- $\boldsymbol{For}(\mathrm{F}\,\varphi)_i = \bigvee_{j=i}^{k} \boldsymbol{For}(\varphi)_j$

Finite Paths – 2



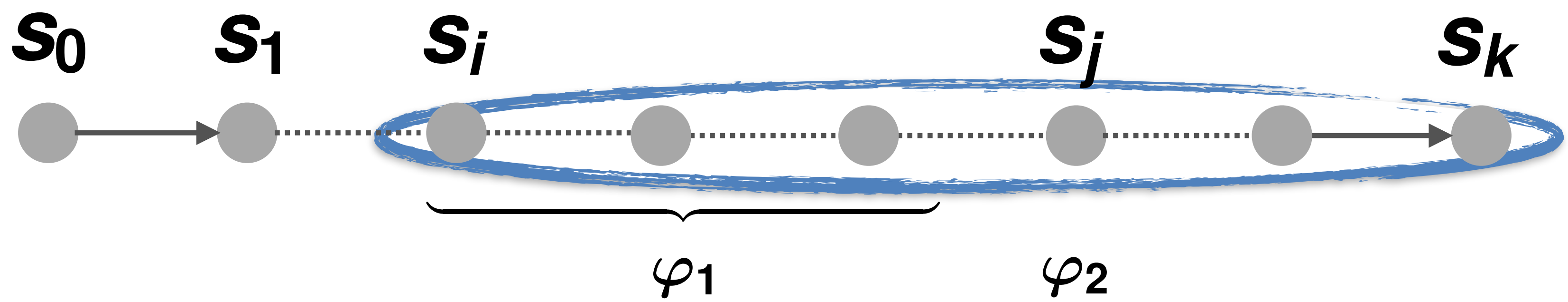
k fixed $\Longrightarrow$ $For(\varphi)$

$\rho \models_k \varphi \iff For(\varphi)$ is SAT $\quad\quad \rho[i] \models_k \varphi \iff For(\varphi)_i$ is SAT

- $For(\mathrm{F}\,\varphi)_i = \bigvee_{j=i}^{k} For(\varphi)_j$

φ_1 **Until** φ_2

Finite Paths – 2

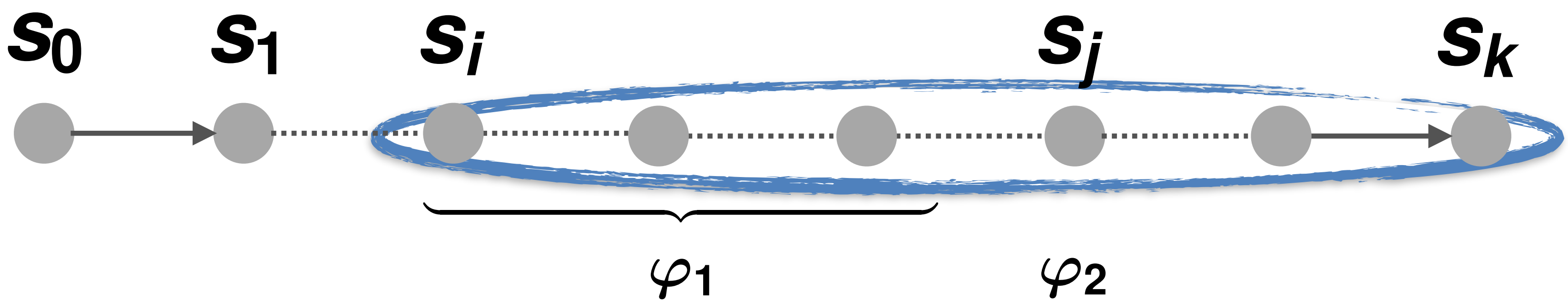


k fixed $\Longrightarrow$ $For(\varphi)$

$\rho \models_k \varphi \iff For(\varphi)$ is SAT $\qquad \rho[i] \models_k \varphi \iff For(\varphi)_i$ is SAT

- $For(\mathrm{F}\,\varphi)_i = \bigvee_{j=i}^{k} For(\varphi)_j$
- $For(\varphi_1 \text{ Until } \varphi_2)_i = \bigvee_{j=i}^{k} \left(For(\varphi_2)_j \wedge \bigwedge_{n=i}^{j-1} For(\varphi_1)_n\right)$

Finite Paths – 2



k fixed $\Longrightarrow$ $\mathit{For}(\varphi)$

$\rho \models_k \varphi \iff \mathit{For}(\varphi)$ is SAT $\qquad \rho[i] \models_k \varphi \iff \mathit{For}(\varphi)_i$ is SAT

- $\mathit{For}(\mathrm{F}\,\varphi)_i = \bigvee_{j=i}^{k} \mathit{For}(\varphi)_j$
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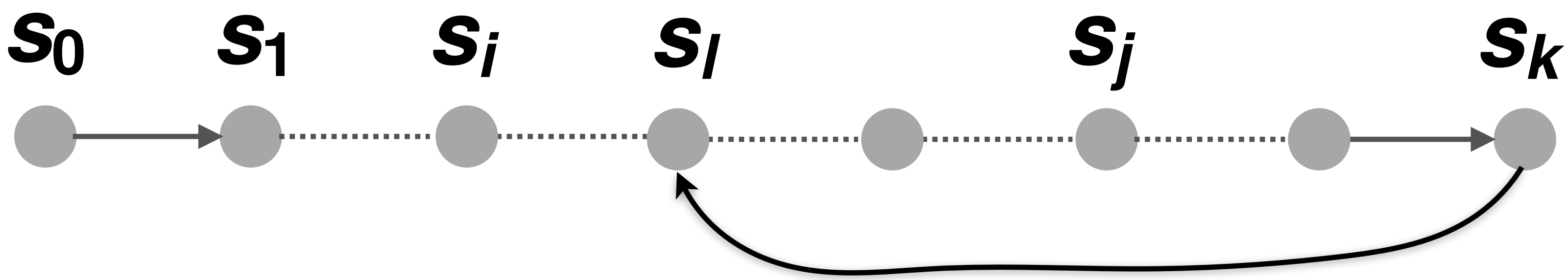
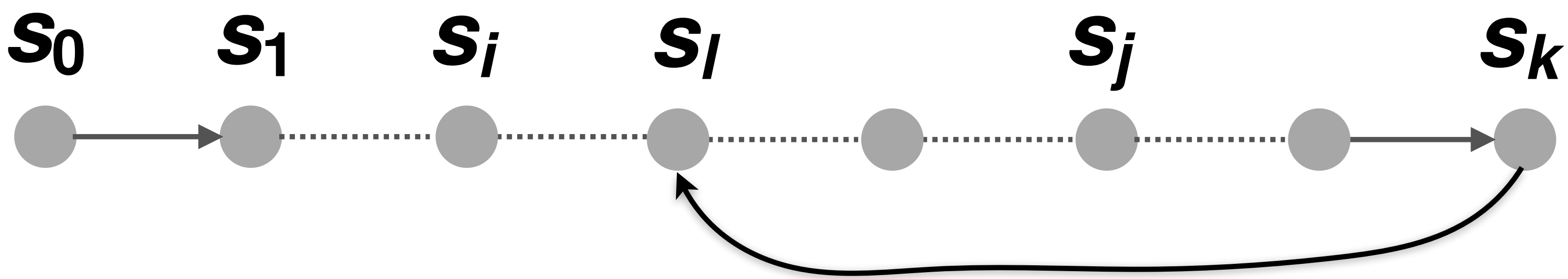
$$\boxed{\mathit{For}(\varphi) = \mathit{For}(\varphi)_0}$$

Infinite Paths – 1

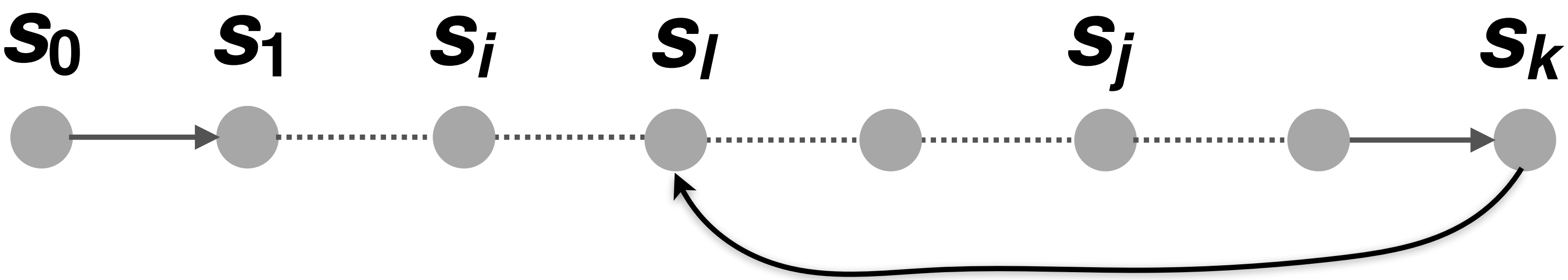


Infinite Paths – 1



$\rho[i] \models_k \varphi \iff \mathit{For}(\varphi)_i^l$ is SAT

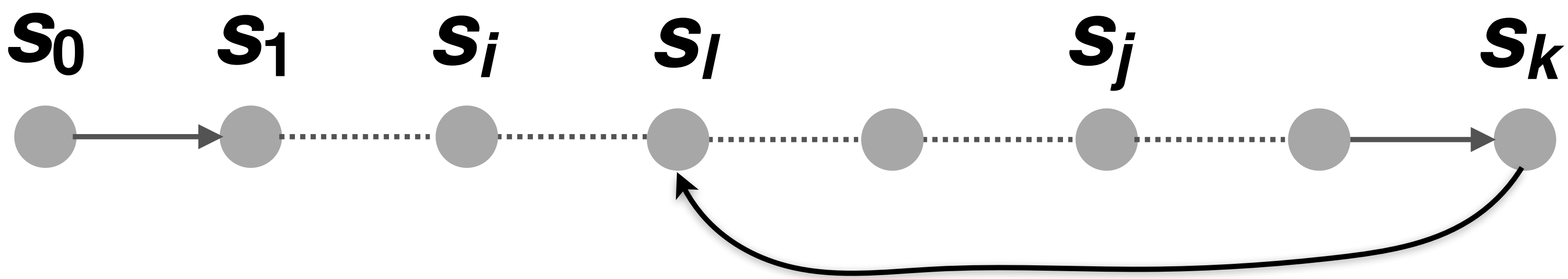
Infinite Paths – 1



$\rho = s_0 \cdots s_{l-1}(s_l \cdots s_k)^\omega$

$\rho[i] \models_k \varphi \iff For(\varphi)_i^l$ is SAT

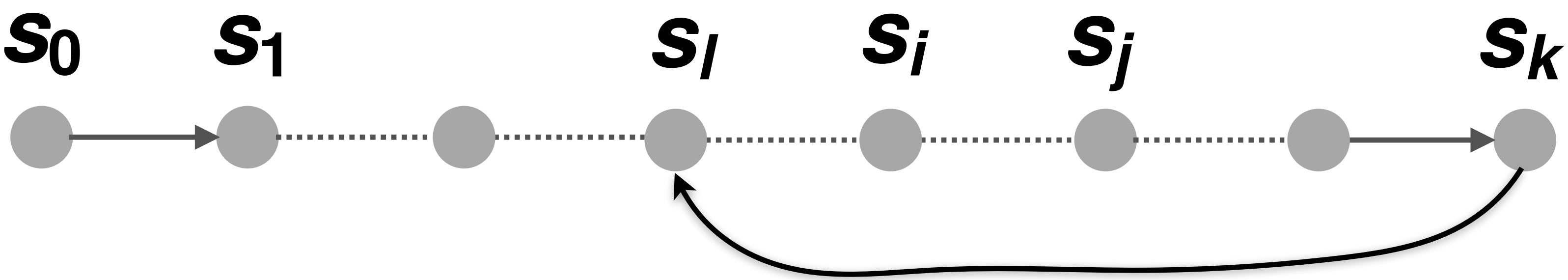
Infinite Paths – 1



$\rho = s_0 \cdots s_{l-1}(s_l \cdots s_k)^\omega$ $\qquad \rho[i] \models_k \varphi \iff For(\varphi)_i^l$ is SAT

- $P(s)$ propositional formula: $For(P(s))_i^l = P(s_i)$
- $For(\varphi_1 \wedge \varphi_2)_i^l = For(\varphi_1)_i^l \wedge For(\varphi_2)_i^l$
- $For(\mathrm{G}\varphi)_i^l = \bigwedge_{j=min(i,l)}^{k} For(\varphi)_j^l$
- $For(\mathrm{F}\varphi)_i^l = \bigvee_{j=min(i,l)}^{k} For(\varphi)_j^l$

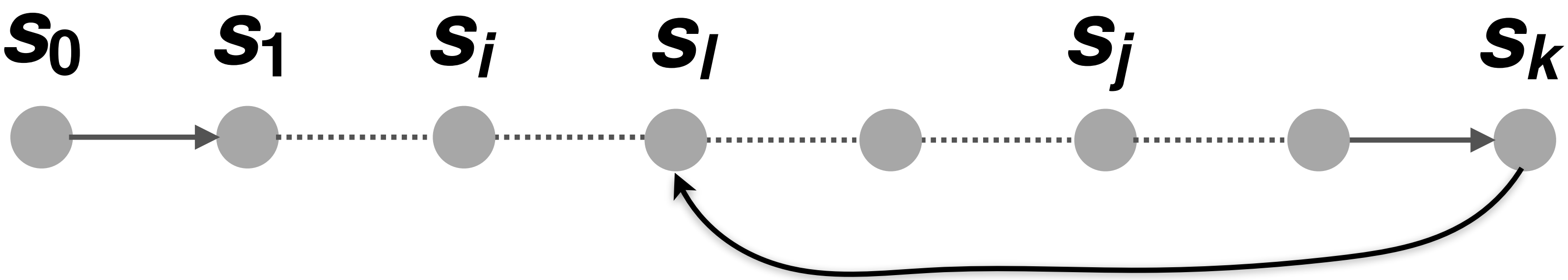
Infinite Paths – 1



$\rho = s_0 \cdots s_{l-1}(s_l \cdots s_k)^\omega$ $\qquad$ $\rho[i] \models_k \varphi \iff For(\varphi)^l_i$ is SAT

- $P(s)$ propositional formula: $For(P(s))^l_i = P(s_i)$
- $For(\varphi_1 \wedge \varphi_2)^l_i = For(\varphi_1)^l_i \wedge For(\varphi_2)^l_i$
- $For(\mathrm{G}\varphi)^l_i = \bigwedge^k_{j=min(i,l)} For(\varphi)^l_j$
- $For(\mathrm{F}\varphi)^l_i = \bigvee^k_{j=min(i,l)} For(\varphi)^l_j$

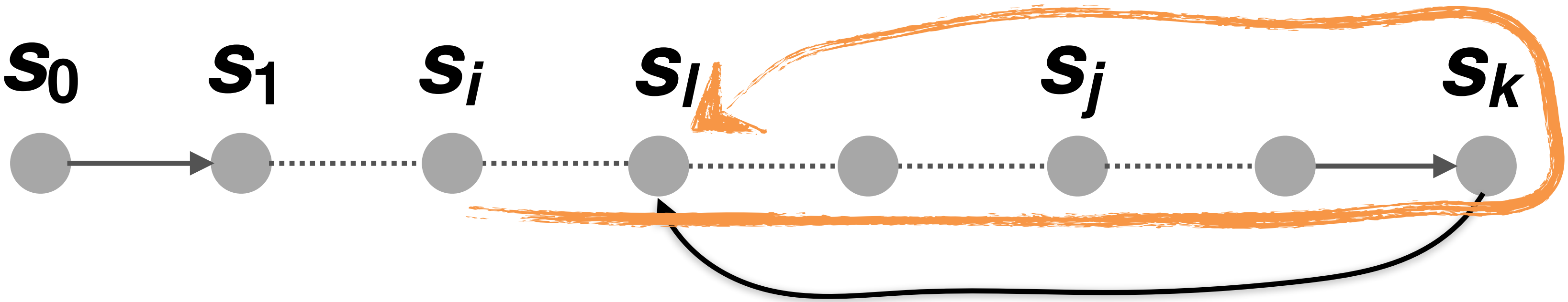
Infinite Paths – 2



$\rho = s_0 \cdots s_{l-1}(s_l \cdots s_k)^\omega$ $\rho[i] \models_k \varphi \iff For(\varphi)^l_i$ is SAT

- $For(\mathrm{X}\,\varphi)^l_i = For(\varphi)^l_{\mathrm{succ}(i)}$
- $For(\varphi_1 \text{ Until } \varphi_2)^l_i =$

Infinite Paths – 2



$\rho = s_0 \cdots s_{l-1}(s_l \cdots s_k)^\omega$ $\qquad$ $\rho[i] \models_k \varphi \iff \mathit{For}(\varphi)_i^l$ is SAT

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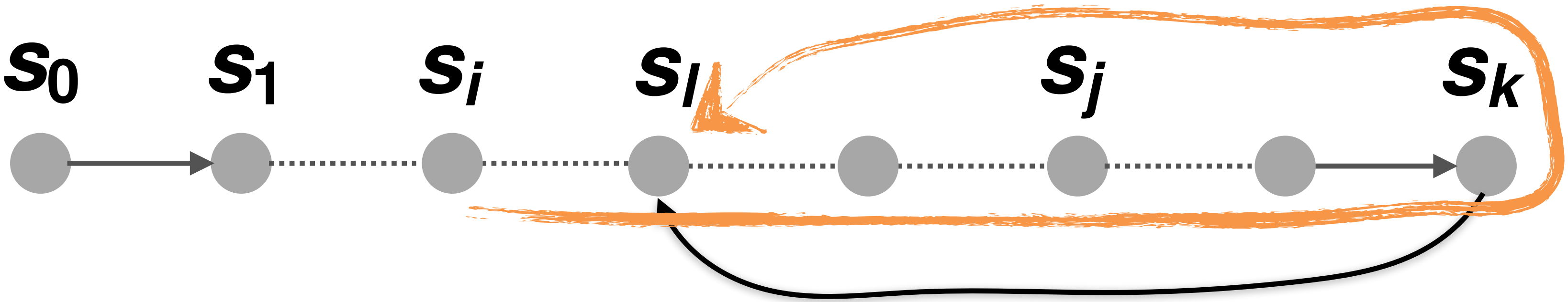
Infinite Paths – 2



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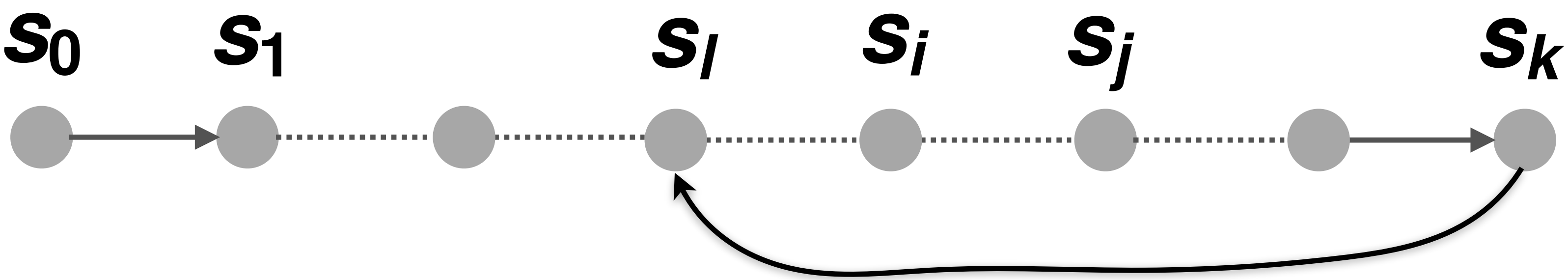
Infinite Paths – 2



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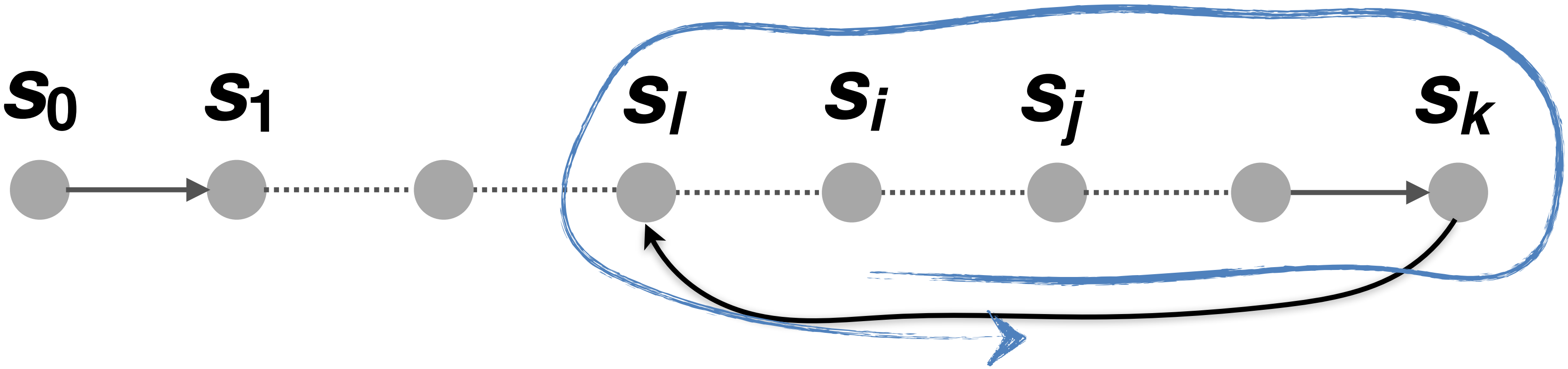
Infinite Paths – 2



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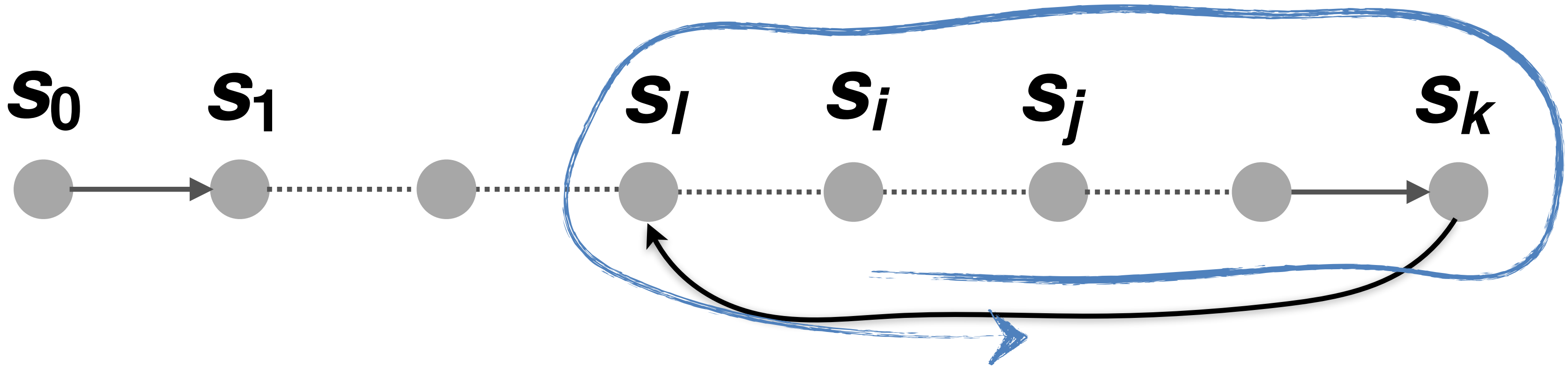
Infinite Paths – 2



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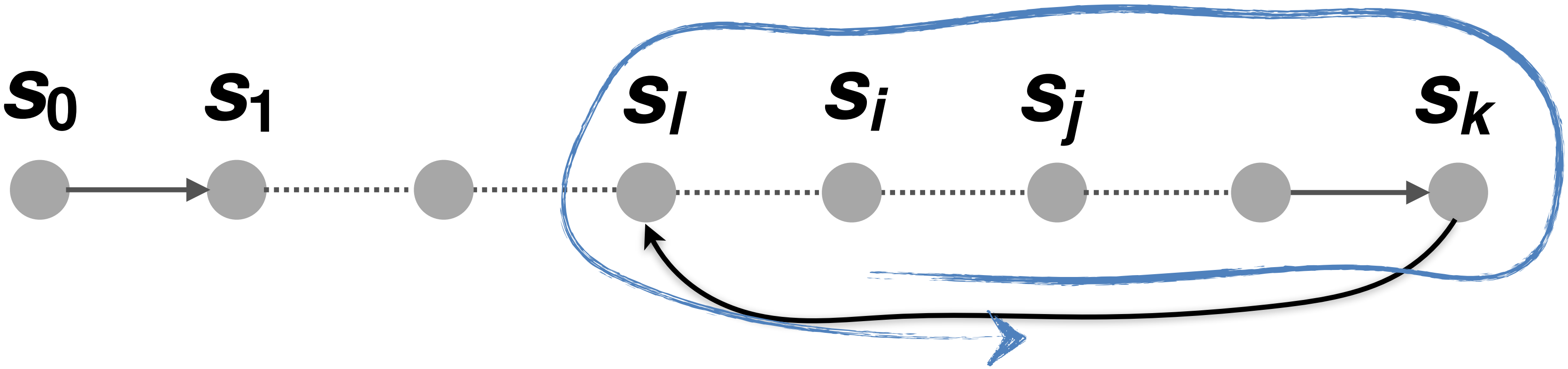
  $\vee \bigvee_{j=l}^{i-1} \left(For(\varphi_2)^l_j \wedge \bigwedge_{n=i}^{k} For(\varphi_1)^l_n \wedge \bigwedge_{n=l}^{j-1} For(\varphi_1)^l_n\right)$

Infinite Paths – 2



$\rho = s_0 \cdots s_{l-1}(s_l \cdots s_k)^\omega$ $\qquad$ $\rho[i] \models_k \varphi \iff For(\varphi)^l_i$ is SAT

- $For(\mathrm{X}\,\varphi)^l_i = For(\varphi)^l_{\mathrm{succ}(i)}$
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$$\boxed{For(\varphi) = For(\varphi)^l_0}$$

Encoding of LTL Formula

Given $\boldsymbol{A} = (\{\mathbf{0}, \mathbf{1}\}^{n}, \boldsymbol{q}_0, \delta)$

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State = vector $(s[0], s[1], \cdots, s[n-1])$

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Encoding of formula φ

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Encoding of formula φ

finite paths

$\neg LOOP \wedge For(\varphi)_0$

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Encoding of formula φ

$$\underbrace{\overset{\textbf{finite paths}}{\neg LOOP \wedge For(\varphi)_0} \quad \bigvee \quad \overset{\textbf{infinite paths}}{\bigvee_{l=0}^{k} LOOP(k,l) \wedge For(\varphi)_0^l}}_{For(\varphi)_k}$$

Model Encoding

Given $\boldsymbol{A} = (\{0, 1\}^n, q_0, \delta)$

Initial states: formula $\boldsymbol{I(s)}$

Transition relation: $\boldsymbol{T(s, s') = \bigvee_{(s,s')\in\delta} f_\delta(s, s')}$

Model Encoding

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Encoding of paths $\leq k + 1$ states

$$For(A) = I(s_0) \wedge \bigwedge_{i=0}^{k-1} T(s_i, s_{i+1})$$

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Encoding of paths $\leq k + 1$ states

$$For(A) = I(s_0) \wedge \bigwedge_{i=0}^{k-1} T(s_i, s_{i+1})$$

3 bit shift register

$x[2]$	$x[1]$	$x[0]$
1	1	0

$I(x) = True$

$T(x, x') \stackrel{def}{=} x'[0] = x[1] \wedge x'[1] = x[2] \wedge x'[2] = 1$

Summary

$$\boldsymbol{A} \models_{\boldsymbol{k}} \mathrm{E}\varphi \iff \boldsymbol{For(A)} \wedge \boldsymbol{For(\varphi)_k} \text{ is SAT}$$

$$\boldsymbol{A} \models \mathrm{A}\neg\varphi \iff \forall \boldsymbol{k} \in \mathbb{N}, \boldsymbol{For(A)} \wedge \boldsymbol{For(\varphi)_k} \text{ is UNSAT}$$

Properties of LTL Bounded Semantics

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P_1: $\rho \models_k \varphi \implies \rho \models \varphi$

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Completeness for Existential Model Checking

$\boldsymbol{P_2}$: if $\boldsymbol{A} \models \mathsf{E}\varphi \implies \exists \boldsymbol{k} \in \mathbb{N}, \boldsymbol{A} \models_{\boldsymbol{k}} \mathsf{E}\varphi$

Try $\boldsymbol{k} = \mathbf{0, 1, 2, 3, \cdots}$,

Completeness for Existential Model Checking

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Try $\boldsymbol{k} = \mathbf{0, 1, 2, 3, \cdots,}$

If $\boldsymbol{A} \not\models_{\boldsymbol{k}} \mathrm{E}\varphi$ for $\boldsymbol{k}$ sufficiently large

can we conclude that $\boldsymbol{A} \not\models \varphi$?

Proof

$\boldsymbol{P_2}$: if $\boldsymbol{A} \models \mathrm{E}\varphi \implies \exists \boldsymbol{k} \in \mathbb{N}, \boldsymbol{A} \models_{\boldsymbol{k}} \mathrm{E}\varphi$

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$\boldsymbol{B_\varphi}$ Büchi automaton for φ

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$$\boldsymbol{A} \times \boldsymbol{B}_{\varphi}$$

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$$\emptyset \neq \mathcal{L}(\boldsymbol{A} \times \boldsymbol{B}_\varphi) \iff \boldsymbol{A} \models \mathrm{E}\varphi$$

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Proof

P_2: if $A \models \mathrm{E}\varphi \implies \exists k \in \mathbb{N}, A \models_k \mathrm{E}\varphi$

$$A \times B_\varphi$$

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$$\emptyset \neq \mathcal{L}(A \times B_\varphi) \iff \begin{cases} u.v^\omega \in \mathcal{L}(A \times B_\varphi) \\ |u| + |v| \leq |A \times B_\varphi| \end{cases}$$

$$|A \times B_\varphi| = |A| \cdot 2^{|\varphi|}$$

Proof

$\boldsymbol{P_2}$: if $\boldsymbol{A} \models \mathrm{E}\varphi \implies \exists \boldsymbol{k} \in \mathbb{N}, \boldsymbol{A} \models_{\boldsymbol{k}} \mathrm{E}\varphi$

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$$|\boldsymbol{A} \times \boldsymbol{B}_\varphi| = |\boldsymbol{A}| \cdot \mathbf{2}^{|\varphi|}$$

Let $\boldsymbol{k} = |\boldsymbol{A}| \cdot \mathbf{2}^{|\varphi|}$, then $\boldsymbol{A} \models_{\boldsymbol{k}} \mathrm{E}\varphi$

Towards Completeness

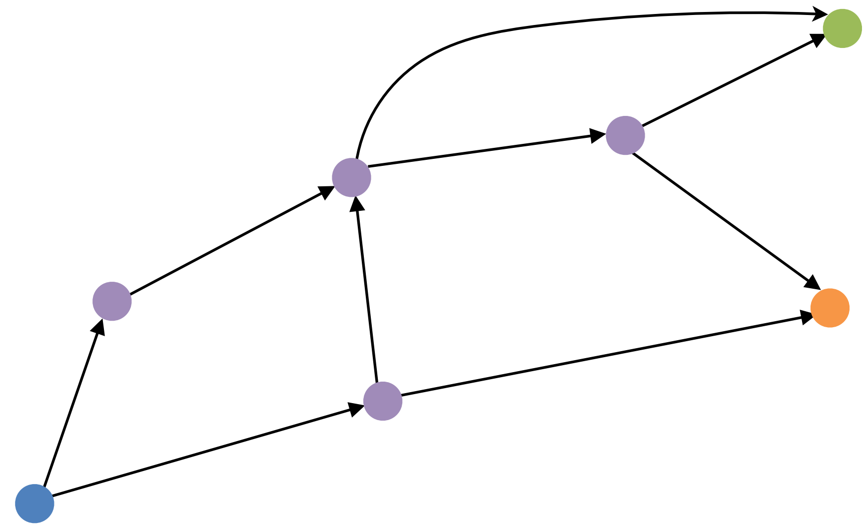
Focus on F$\boldsymbol{p}$, $\boldsymbol{p}$ atomic proposition

Computing a better bound

Towards Completeness

Focus on F***p***, ***p*** atomic proposition

Computing a better bound



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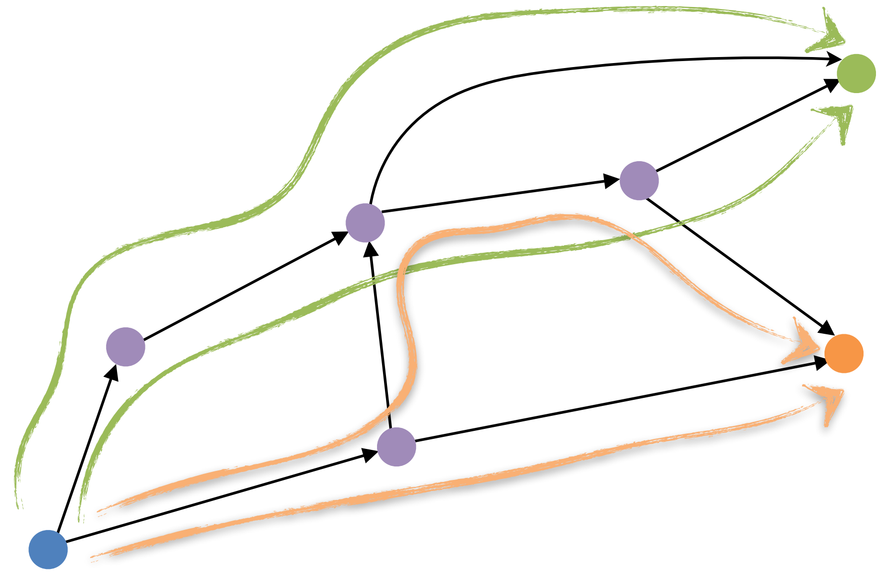

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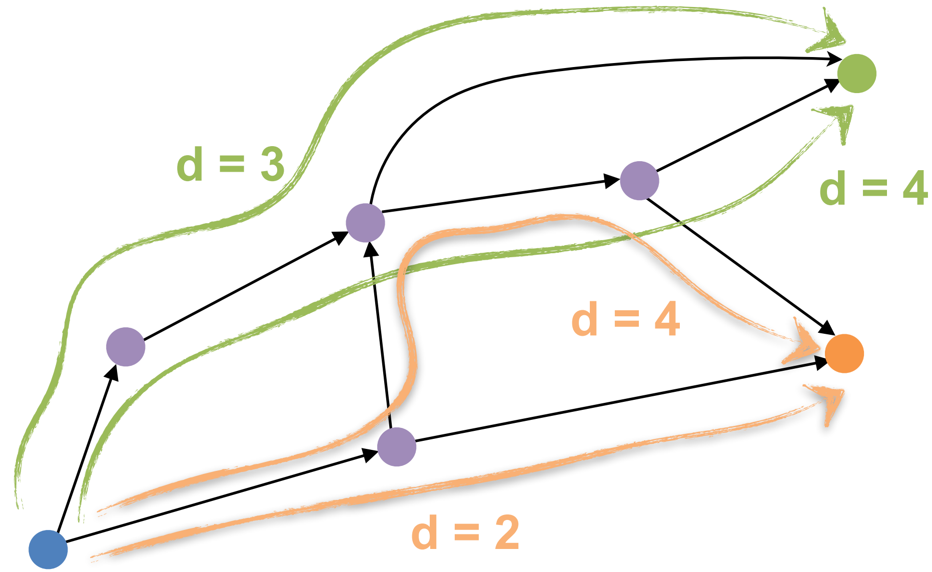

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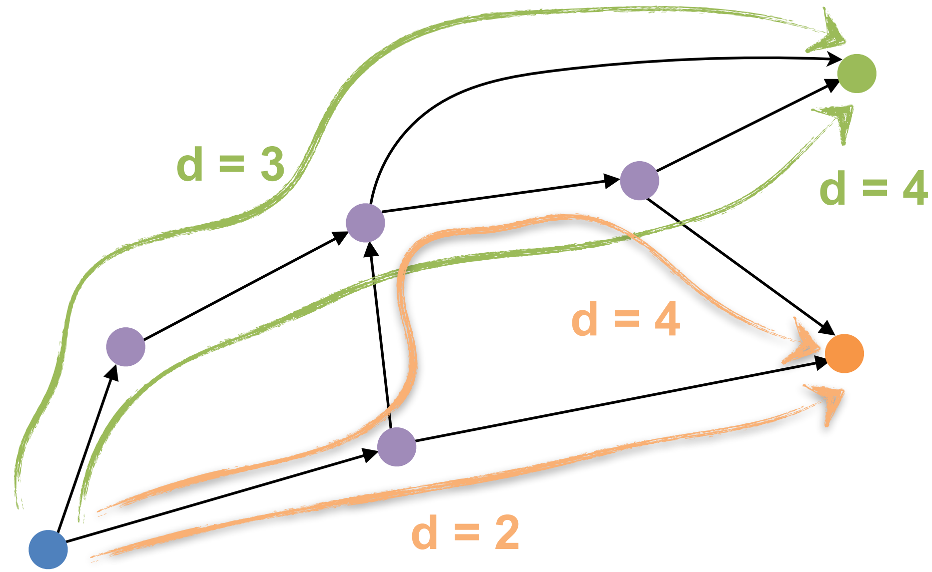
Diameter = 3



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$\boldsymbol{s} \models \boldsymbol{p}$ reachable $\implies$ $\boldsymbol{s}$ reachable on a shortest path

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$$\boldsymbol{s} \models \boldsymbol{p} \text{ reachable} \implies \boldsymbol{s} \text{ reachable on a shortest path}$$

$$\boldsymbol{A} \models \mathrm{EF}\boldsymbol{p} \iff \boldsymbol{A} \models_{\boldsymbol{k}} \mathrm{EF}\boldsymbol{p} \text{ for some } \boldsymbol{k} \leq \mathrm{Diameter}(\boldsymbol{A})$$